

Do Agency Contracts Facilitate Upstream Collusion?*

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This version: February 8, 2019

Abstract

This paper presents a study of whether agency contracts facilitate collusion among upstream manufacturers, as compared to traditional wholesale contracts. We consider an infinitely repeated game with a monopoly platform and multiple manufacturers. Our analysis shows that the critical discount factor, above which the upstream collusion can be sustainable by Nash-reversion trigger strategies, is the same under wholesale and agency contracts. That result indicates that the agency contract is not necessarily anticompetitive. By contrast, in an extended model with competing platforms, we show that the agency contract facilitates upstream collusion because accepting it under the agency contract mitigates platform competition.

Keywords: *agency contract, cartel stability, upstream collusion, wholesale contract*

JEL Classification: L13, L42, L81, D21, D43, D86, K21

*Special appreciation is extended to Jay Pil Choi, Daisuke Hirata, Akifumi Ishihara, Toshihiro Matsumura, Noriaki Matsushima, Hodaka Morita, and Naoki Watanabe. The authors also thank seminar participants at Hitotsubashi University and Kyoto University, and participants at the 2018 annual conference of the JAAE in Kyoto and the 2018 Annual Meeting of the APIOC in Melbourne for their helpful comments. This work was supported by JSPS KAKENHI Grant Numbers 16K17126, 17H00959, and 18J10212.

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1 Introduction

In March 2016, the U.S. Supreme Court denied a hearing an appeal lodged by Apple Inc., leaving the conclusion that Apple violated *Section 1 of the Sherman Act* by orchestrating a horizontal conspiracy among five of the six largest U.S. book publishers (Hachette, HarperCollins, Macmillan, Penguin, and Simon & Schuster) to raise e-book prices.¹ The U.S. Department of Justice (DOJ) initially filed its complaint that *agency agreements* played an instrumental role.² Along with the agency contract, Apple organized a price cartel among the publishers by interacting and sharing information with them. That was a *per se* violation of the Sherman Act. However, it is noteworthy that the district court conceded that the agency agreement is not inherently illegal, and is an entirely lawful contract.

In the case of the e-book cartel, because Apple assembled “a horizontal price-fixing conspiracy” consisting of a group of competitors, it was a *per se* violation of the Sherman Act. Moreover, the district court concluded that even if this case were analyzed under *the rule of reason*, it would still constitute an unreasonable restraint of trade in violation of §1. However, it has not been evaluated whether the agency contract itself facilitates collusion among upstream publishers. This paper provides an economic explanation for when and how the agency contract facilitates collusion among publishers.

To this end, we first develop a stylized model of an infinitely repeated game comprising a monopoly platform and multiple manufacturers that sell differentiated products via the platform. The manufacturers can be interpreted as publishers in the example of the e-book market given above. Other examples include e-commerce platform markets (many manufacturers’ products are sold on online platforms such as Amazon, eBay, Rakuten in Japan, and Taobao in China) and mobile application markets (application developers distribute their “apps” through Apple App Store and Google Play Store).

We investigate two contract forms: a *wholesale contract* and an *agency contract*, which are the two simplest and most popular contracts adopted by platforms.³ In the stage-game, for each contract, the monopoly platform and manufacturers move sequentially explained below. Under the wholesale contract, each manufacturer chooses its wholesale price in the first round before the platform sets the retail prices in the second round.⁴ Under the agency contract, the platform chooses a revenue-sharing rule (a so-called royalty rate or commission rate) in the first round before each manufacturer sets the retail price directly in the second round. For each contract,

¹See *U.S. v. Apple, Inc.* for more details.

²This agency agreement was a contract between a platform and publishers, by which the publishers can set final retail prices. Sales revenue obtained through the platform is split between publishers and the platform subject to a fixed revenue-sharing rule. Recently, such agency contracts have been used widely on other online platforms as well as e-book platforms.

³In mobile application markets, agency contracts have been most widely used. By contrast, both the wholesale and agency contracts are widely used, in e-commerce markets. For example, according to Amazon’s full-year 2017 financial results, net sales from online stores (i.e., the wholesale model) were \$108.4 billion, whereas those from third-party seller services (i.e., the agency model) were \$31.9 billion.

⁴To avoid duplication of words, we use ‘*round*’ to designate each stage in the stage-game.

we obtain the value of the critical discount factor, above which joint profit maximizing collusion among manufacturers is sustainable by Nash-reversion trigger strategies. Then, we compare the two critical discount factors to assess whether the agency contract facilitates upstream collusion, or not. We can say that the agency contract facilitates upstream collusion if it reduces the critical discount factor.

In this paper, we focus on pure strategy Markov perfect equilibria. A technical issue exists in obtaining the critical discount factor under the agency contract because the platform takes its action before the cartel party (i.e., manufacturers) in every stage game. By virtue of its prior action, the platform can strategically manipulate the sustainability of upstream collusion. Therefore, to keep the platform from actively engaging in collusion, we confine our attention to the equilibria under which the platform follows Markov strategies. If there are multiple pure strategy equilibria, then we select the platform-preferred equilibria, that maximize the payoff of the platform. Within this class of equilibria, we define the critical discount factor as the smallest discount factor above which upstream collusion can be sustainable.

Our analysis of the monopoly platform case shows that the critical discount factor is the same under both wholesale and agency contracts. In other words, the type of contract is independent of whether upstream collusion arises. This correspondence also holds even if the monopoly platform sets a revenue-sharing rule at first and commits to it, and manufacturers play an infinitely repeated game. Rather, it would stem from the Nash-reversion trigger strategy we adopt. We further investigate the monopoly platform case with the optimal punishment strategy in the sense of Abreu (1986). Furthermore, we demonstrate that the agency contract can facilitate upstream collusion. In this respect, the agency contract can be anticompetitive. At the same time, however, given upstream collusion to be sustained, the agency contract generates higher consumer surplus and social welfare compared to the wholesale contract. In summary, the agency contract can not necessarily be characterized as anticompetitive in markets with a monopoly platform.

In addition to the monopoly platform model described above, we examine the extended model with two competing platforms. Under the agency contract, the competing platforms must determine their actions (i.e., revenue-sharing rules) with consideration not only of the effects on manufacturers but also the effects on rival platforms, which makes it difficult to set high revenue-sharing rules. By contrast, upstream collusion increases gross revenues in the channel; it then heightens unilateral incentives of platforms to raise their revenue-sharing rules. In other words, accepting upstream collusion can mitigate platform competition.

As a result, we demonstrate that the agency contract facilitates upstream collusion in the markets with competing platforms, which is a sharp contrast to the result derived using the monopoly platform model. Moreover, introducing agency contracts not only facilitates upstream collusion, but also provokes coordination between competing platforms, which makes it possible to enhance their revenue-sharing rules. In fact, in mobile application markets with competition between Apple App Store and Google Play Store, although developers set the prices of their apps, both Apple and

Google impose a 30% commission rate on each purchase.⁵ That high commission rate led to a key antitrust case that reached the U.S. Supreme Court in November 2018.

The results derived and presented herein represent several important contributions to competition policy. First, the agency contract itself is not anticompetitive: when the market is served by a monopoly platform, the agency contract neither facilitates nor obstructs upstream collusion. Rather, given upstream collusion, the agency contract engenders higher consumer surplus and social welfare compared to the wholesale contract. By contrast, with platform competition, the introduction of agency contracts has an anticompetitive concern to facilitate upstream collusion, and to mitigate platform competition. Therefore, strong revenue-sharing rules should be monitored carefully along with the formation of upstream collusion, which is the second and the most important message of the paper.

1.1 Case of e-book cartel

According to *U.S. v. Apple, Inc.*, we summarize the case of the e-book cartel in relation to the purpose of this paper.⁶

Amazon, a famous e-book platform, started selling its *Kindle* e-book reader in 2007. First, Amazon signed wholesale contracts with e-book publishers, whereby each publisher sets a wholesale price of its e-book before Amazon charges retail prices for those e-books. In practice, Amazon charged \$9.99 for certain new releases and bestselling e-books. When Apple entered the e-book market with its new *iPad* device in 2010, it convinced publishers to adopt agency agreements by which publishers can control the prices of their e-books, with Apple receiving a 30% commission, to break Amazon's monopolistic grip on the market. In addition, Apple included a *most-favored-nation (MFN) clause* in their contract with major publishers, which allowed Apple to sell e-books at its competitors' lowest price (i.e., Amazon's \$9.99). The existence of the MFN clause forced publishers to negotiate the signing of an agency contract with Amazon.⁷ In consequence, Amazon also moved to the agency contract.

The industrial movement toward agency contracts engendered an increase in e-book retail prices of about 18%, on average. Particularly, the price of *New York Times* bestsellers rose by about 40%. *What caused such an increase in the retail prices of e-books?* On September 5, 2013, the district court issued a final injunctive order prohibiting Apple from enforcing the MFN clause with e-book publishers and requiring Apple to modify its agency agreements. However, it is noteworthy that the district court conceded that both the agency agreement and MFN clause are not inherently illegal. Therefore, in this case, the increase in e-book retail prices might have resulted from another cause. The district court found evidence indicating that the publishers joined in a horizontal price-

⁵In 2016, Apple introduced a new revenue-sharing rule, such that the 30% commission rate will drop to 15% for each subscriber active for at least one year. Google also followed this revenue-sharing rule from the beginning of 2018 in a move to compete better with Apple's offering for iOS developers.

⁶Other examples of digital cartels on online platforms include the case of poster cartels and the Lithuanian Eturas case (Rusu, 2016).

⁷The analyses presented in this paper do not incorporate the MFN clause into the model, instead emphasizing how the contract structure affects the sustainability of the upstream price cartel.

fixing conspiracy, undertaken by Apple, to increase the retail price of e-books by eliminating price competition. Apple's central role in the conspiracy was proven as a *per se* violation of the Sherman Act.

1.2 Literature

After an increase in e-book prices associated with Apple's entry into the e-book market in 2010, studies of whether agency agreements raise the retail prices were launched. A seminal work is that of Johnson (2017), who shows that the shift from the wholesale model to the agency model lowers retail prices, which benefits retailers and consumers, but which harms suppliers. Gaudin and White (2014) investigate the bilateral monopoly model with an upstream e-book publisher and a downstream retailer. Based on their findings, they show that the wholesale contract engenders a higher retail price if and only if the elasticity of demand strictly decreases as quantity increases. Additionally, they consider an extended case in which the retailer has monopoly market power on an e-book reader device that consumers must purchase to use e-books. In this case, under the wholesale contract, the retailer sets the e-book price at the wholesale price it pays to the publisher and extracts residual surplus through the device price (i.e., two-part tariff pricing). Consequently, no double marginalization problem exists. By contrast, under the agency contract, the double marginalization problem arises because the retailer charges a positive commission fee if the elasticity of demand strictly decreases as quantity increases. As a result, the agency contract can engender a higher retail price.

Foros et al. (2017) consider platform competition with a bilateral duopoly market consisting of two upstream suppliers and two downstream platforms. In their model, under both wholesale and agency contracts, sales revenues are shared between the upstream and the downstream firms. They show that the agency contract engenders higher retail prices than the wholesale contract if and only if the degree of substitution between the platforms is high compared to the degree of substitution between the suppliers. Lu (2017) also analyzes the bilateral duopoly model to demonstrate the pro-competitive effect of agency contracts. He shows that the retail prices are lower and the demands are greater under the agency contract than under the wholesale contract, i.e., consumers benefit from the agency contract. The difference from the model of Foros et al. (2017) is the setting of the wholesale contract. He assumes that the upstream firms charge their linear wholesale prices before the downstream firms choose the retail prices, unlike the revenue-sharing contract in Foros et al. (2017). The setting of the stage-game in this paper is similar to that of Lu (2017).

The present paper is also related to the literature on tacit collusion in vertically related markets (e.g., Bernheim and Whinston, 1985; Cooper, 1986; Neilson and Winter, 1993).⁸ More recently, Jullien and Rey (2007) show that retail-price maintenance (RPM) agreements can facilitate collusion among upstream manufacturers when they observe only the resultant retail prices (i.e., every manufacturer cannot observe wholesale prices set by the other rivals). The RPM agreements im-

⁸The framework used by Bernheim and Whinston (1985) and Cooper (1986) is a finite horizon; it is not what we mean by collusion in our manuscript.

prove the detectability of a manufacturer’s deviation by alleviating the flexibility of retail pricing, which makes punishment easier.

Nocke and White (2007) examine the effects of vertical mergers on the sustainability of upstream collusion. They elucidated that a vertical merger has an *outlet effect* and a *punishment effect*. The outlet effect is that the integrated firm can reject offers from other deviating upstream firms, which facilitates collusion. The punishment effect is that the integrated firm can receive larger profits in the non-cooperative phase, thereby making it easy for the integrated firm to deviate from the collusion. The relative magnitude of these two effects determines whether the vertical merger facilitates upstream collusion. Reisinger and Thomes (2017) compare cartel sustainability among manufacturers under various channel structures. They show that the collusion among manufacturers is more sustainable when they compete via independent retailers than when they compete via a common retailer. The reason is as follows: under the common retailer model, a manufacturer can defect from the collusion to induce the common retailer to reject the non-deviating manufacturers offer, thereby monopolizing the retail market and generating a large deviation payoff.

In those earlier studies, cartel parties determine their joint profit maximizing actions before other players move. In other words, no player strategically designs its own actions to influence cartel formation. One notable exception is work by Huang (2017), who studies downstream collusion in a vertically related market with an upstream supplier and two downstream retailers. The downstream retailers engage in collusive actions infinite-repeatedly after the supplier offers a stationary two-part tariff contract to the retailers. Consequently, in her model, the upstream supplier can strategically design its supply contract in an effort to influence collusion between the downstream retailers. A similar situation is investigated in Section 4.1 of this paper, where the platform commits to a fixed revenue-sharing rule before manufacturers play an infinitely repeated game. Furthermore, in the main analysis of this paper, we examine the more general model by which, in every stage-game, the platform determines a revenue-sharing rule before the manufacturers collude to maximize their joint profit, which implies that the platform can foster or hinder the cartel formation more dynamically.

Nevertheless, no study described in the relevant literature addresses the question of whether agency contracts facilitate upstream collusion as compared to traditional wholesale contracts. This study contributes to the literature by showing when and how the agency contract facilitates upstream collusion. The results obtained from this study can present important policy implications.

2 Monopoly Platform

As described in this section, we examine a model with a monopoly platform and multiple manufacturers. Section 2.1 explains analysis of the stage-game. Section 2.2 presents examination of an infinitely repeated game to derive the critical discount factors under the wholesale and agency contracts; then they are compared to show whether the agency contract facilitates upstream collusion, or not.

2.1 The stage-game

In this subsection, we describe the setting of the stage-game; then we derive the static equilibrium and the collusive equilibrium, respectively, under the wholesale and agency contracts. Lastly, those stage-game outcomes are compared.

Setting of the stage-game We consider a vertically related market with a monopoly platform P and n manufacturers ($n \geq 2$). Each manufacturer i ($i = 1, \dots, n$) produces good i and sells it through platform P . Let p_i and q_i be the price and the quantities of good i sold on the platform. The utility function of a representative consumer is given as

$$u(q_1, \dots, q_n) = \sum_i a q_i - \sum_i \frac{b}{2} q_i^2 - \sum_{i \neq j} b \theta q_i q_j - \sum_i p_i q_i, \quad (1)$$

which yields the inverse demand function as $p_i = a - b q_i - b \theta \sum_{j \neq i} q_j$. Solving those inverse demand functions with respect to quantity yields the demand function of good i as

$$q_i = \alpha - \beta p_i + \gamma \sum_{j \neq i} p_j, \quad (2)$$

where $\alpha \equiv \frac{a}{b\{1+(n-1)\theta\}}$, $\beta \equiv \frac{\{1+(n-2)\theta\}}{b(1-\theta)\{1+(n-1)\theta\}}$, and $\gamma \equiv \frac{\theta}{b(1-\theta)\{1+(n-1)\theta\}}$. We let $\sigma \equiv \beta - (n-1)\gamma > 0$, which means that the own-price effect dominates the sum of cross-price effects.

This paper presents examination of two contract forms: wholesale and agency. The stage-game of each contract is presented as follows. Under the wholesale contract, each manufacturer i sets its wholesale price w_i in the first round before platform P charges a pair of retail prices (p_1, \dots, p_n) in the second round. We assume that manufacturers incur a constant marginal cost of production $c > 0$. The profits of manufacturer i and of platform P are given respectively as $\pi_i = (w_i - c)q_i$ and $\Pi = \sum_i (p_i - w_i)q_i$. In contrast, under the agency contract, the platform offers a revenue-sharing rule $s \in [0, 1]$ for all manufacturers in the first round before each manufacturer i sets its retail price p_i in the second round. The profits of manufacturer i and of platform P are given respectively as $\pi_i = \{(1-s)p_i - c\}q_i$ and $\Pi = \sum_i s p_i q_i$.

In the symmetric equilibrium (i.e., $p_i = p$ and $q_i = q$ for all i), consumer surplus is given as $CS = nq^2/(2\sigma)$. Additionally, we interpret social welfare as a sum of consumer surplus and profits of all firms, i.e., $SW = CS + \Pi + \sum_i \pi_i$.

Stage-game analysis under the wholesale contract One can consider the situation in which the game above is played only once under the wholesale contract. The stage-game is solved using backward induction. In the second round, given a pair of wholesale prices $w = (w_1, \dots, w_n)$, platform P chooses retail prices (p_1, \dots, p_n) to maximize its profit. Solving $\partial \Pi / \partial p_i = 0$ for all i implies that $p_i(w_i) = \alpha / (2\sigma) + w_i / 2$.

Table 1: Stage-game equilibrium and collusive equilibrium under the wholesale contract

| $k = \{N, C\}$ | Stage-game Nash equilibrium (N) | Collusive equilibrium (C) |
|----------------|--|---|
| w_{Wk}^* | $\frac{\alpha + \beta c}{\beta + \sigma}$ | $\frac{\alpha}{2\sigma} + \frac{c}{2}$ |
| p_{Wk}^* | $\frac{\alpha}{2\sigma} + \frac{\alpha + \beta c}{2(\beta + \sigma)}$ | $\frac{3\alpha + \sigma c}{4\sigma}$ |
| q_{Wk}^* | $\frac{\beta(\alpha - \sigma c)}{2(\beta + \sigma)}$ | $\frac{\alpha - \sigma c}{4}$ |
| π_{Wk}^* | $\frac{\beta(\alpha - \sigma c)^2}{2(\beta + \sigma)^2}$ | $\frac{(\alpha - \sigma c)^2}{8\sigma}$ |
| Π_{Wk}^* | $n \cdot \frac{\beta^2(\alpha - \sigma c)^2}{4\sigma(\beta + \sigma)^2}$ | $n \cdot \frac{(\alpha - \sigma c)^2}{16\sigma}$ |
| CS_{Wk}^* | $n \cdot \frac{\beta^2(\alpha - \sigma c)^2}{8\sigma(\beta + \sigma)^2}$ | $n \cdot \frac{(\alpha - \sigma c)^2}{32\sigma}$ |
| SW_{Wk}^* | $n \cdot \frac{\beta(3\beta + 4\sigma)(\alpha - \sigma c)^2}{8\sigma(\beta + \sigma)^2}$ | $n \cdot \frac{7(\alpha - \sigma c)^2}{32\sigma}$ |

Then, the maximization problem faced by manufacturer i in the first round is written as

$$\max_{w_i} \pi_i(w) = (w_i - c) \left(\alpha - \beta p_i(w_i) + \gamma \sum_{j \neq i} p_j(w_j) \right). \quad (3)$$

Solving $\partial \pi_i / \partial w_i = 0$ for all i implies that $w_{WN}^* = (\alpha + \beta c) / (\beta + \sigma)$. We here use subscript W to indicate the *wholesale* contract, subscript N to denote the stage-game *Nash* equilibrium, and superscript ‘*’ to represent the equilibrium result of the monopoly platform model. The equilibrium retail price, profit of manufacturers, profit of the platform, consumer surplus, and social welfare are presented in Table 1.

Next, we derive the collusive stage-game equilibrium. The best response of platform P at the second round is unchanged, i.e., $p_i(w_i) = \alpha / (2\sigma) + w_i / 2$. In the first round, manufacturers collude in their wholesale prices to maximize their joint profit, which is given as $\pi^C = \sum_i \pi_i = \sum_i (w_i - c) q_i$. The maximization problem of cartel party (i.e. manufacturers) is the following.

$$\max_{w_1, \dots, w_n} \pi^C(w) = \sum_i (w_i - c) \left(\alpha - \beta p_i(w_i) + \gamma \sum_{j \neq i} p_j(w_j) \right) \quad (4)$$

Solving this problem yields $w_{WC}^* = \alpha / (2\sigma) + c / 2$. We use subscript C to denote the *collusive* equilibrium. Other outcomes are presented in Table 1.

By comparing the stage-game Nash equilibrium with the collusive result obtained under the wholesale contract, one derives several results indicating how upstream collusion affects the vertical relation.

Lemma 1. *Consider the case with a monopoly platform. Under the wholesale contract, upstream collusion increases the wholesale and retail prices (i.e., $w_{WN}^* < w_{WC}^*$ and $p_{WN}^* < p_{WC}^*$), which decreases the quantities demanded, consumer surplus, and social welfare (i.e., $q_{WN}^* > q_{WC}^*$,*

$CS_{WN}^* > CS_{WC}^*$, and $SW_{WN}^* > SW_{WC}^*$). By colluding, the manufacturers receive greater profit (i.e., $\pi_{WN}^* < \pi_{WC}^*$), although the profit of the platform is decreased (i.e., $\Pi_{WN}^* > \Pi_{WC}^*$).

Proof. The result can be shown using simple calculations. \square

Under the wholesale contract, collusion among manufacturers increases the wholesale price, which in turn induces the platform to choose higher retail prices. The wholesale price increased by the collusion raises the manufacturers' margin per unit sold, which increases their profits. Simultaneously, the resulting higher retail price shrinks the quantities demanded; it thereby decreases the profit of the platform. Considered comprehensively, through the increased retail price, both consumer surplus and social welfare worsen.

Stage-game analysis under the agency contract As in the case of the wholesale contract, one can consider a situation in which the stage-game is played only once under the agency contract. In the second round, each manufacturer i chooses its retail price p_i independently, given the revenue-sharing rule s chosen by the platform. Solving $\partial\pi_i/\partial p_i = 0$ for all i implies $p_{AN}(s) = \left(\alpha + \beta\frac{c}{1-s}\right)/(\beta + \sigma)$, where we use subscript A to denote the *agency* contract. The corresponding quantities, profit of manufacturers, and profit of platform P are given respectively as

$$q_{AN}(s) = \frac{\beta\left(\alpha - \sigma\frac{c}{1-s}\right)}{\beta + \sigma}, \quad (5)$$

$$\pi_{AN}(s) = \beta(1-s)\left(\frac{\alpha - \sigma\frac{c}{1-s}}{\beta + \sigma}\right)^2, \quad (6)$$

$$\Pi_{AN}(s) = \sum_i \left(s \cdot \frac{\alpha + \beta\frac{c}{1-s}}{\beta + \sigma} \cdot \frac{\beta\left(\alpha - \sigma\frac{c}{1-s}\right)}{\beta + \sigma} \right). \quad (7)$$

In the first round, the platform sets the revenue-sharing rule s_{AN}^* which satisfies the first-order condition, which is equivalent to

$$\begin{aligned} \frac{\partial\Pi_{AN}(s_{AN}^*)}{\partial s} &= \frac{n\beta}{(\beta + \sigma)^2} \cdot \left[\begin{aligned} &\left(\alpha + \beta\frac{c}{1-s_{AN}^*}\right)\left(\alpha - \sigma\frac{c}{1-s_{AN}^*}\right) \\ &+ \frac{s_{AN}^*c}{(1-s_{AN}^*)^2} \left\{ \alpha(\beta - \sigma) - 2\beta\sigma\frac{c}{1-s_{AN}^*} \right\} \end{aligned} \right] = 0, \\ \iff &\frac{n\beta}{(\beta + \sigma)^2} \cdot \frac{1}{(1-s_{AN}^*)^3} \cdot \xi(s_{AN}^*) = 0, \end{aligned} \quad (8)$$

where $\xi(s) \equiv \alpha^2(1-s)^3 + \{(\beta - \sigma)\alpha c + \beta\sigma c^2\}(1-s) - 2\beta\sigma c^2$. s_{AN}^* is unique over $s \in (0, 1 - \sigma c/\alpha)$ and satisfies the second-order condition.⁹ The equilibrium outcomes of the stage game are written

⁹ It holds that $\text{sign } \frac{\partial\Pi_{AN}(s)}{\partial s} = \text{sign } \xi(s)$ for all $s \in (0, 1)$. We have

$$\begin{aligned} \xi(0) &= (\alpha + \beta c)(\alpha - \sigma c) > 0, \\ \xi\left(1 - \frac{\sigma c}{\alpha}\right) &= -\frac{c^2\sigma}{\alpha}(\sigma + \beta)(\alpha - \sigma c) < 0, \end{aligned}$$

Table 2: Stage-game equilibrium and collusive equilibrium under the agency contract

| $k = \{N, C\}$ | Stage-game Nash equilibrium (N) | Collusive equilibrium (C) |
|----------------|--|--|
| s_{Ak}^* | s_{AN}^* in equation (8) | s_{AC}^* in equation (12) |
| p_{Ak}^* | $\frac{1}{\beta+\sigma} \left(\alpha + \beta \frac{c}{1-s_{AN}^*} \right)$ | $\frac{1}{2\sigma} \left(\alpha + \sigma \frac{c}{1-s_{AC}^*} \right)$ |
| q_{Ak}^* | $\frac{\beta}{\beta+\sigma} \left(\alpha - \sigma \frac{c}{1-s_{AN}^*} \right)$ | $\frac{1}{2} \left(\alpha - \sigma \frac{c}{1-s_{AC}^*} \right)$ |
| π_{Ak}^* | $\frac{\beta(1-s_{AN}^*)}{(\beta+\sigma)^2} \left(\alpha - \sigma \frac{c}{1-s_{AN}^*} \right)^2$ | $\frac{1-s_{AC}^*}{4\sigma} \left(\alpha - \sigma \frac{c}{1-s_{AC}^*} \right)^2$ |
| Π_{Ak}^* | $n \cdot \frac{\beta s_{AN}^*}{(\beta+\sigma)^2} \left(\alpha + \beta \frac{c}{1-s_{AN}^*} \right) \left(\alpha - \sigma \frac{c}{1-s_{AN}^*} \right)$ | $n \cdot \frac{s_{AC}^*}{4\sigma} \left(\alpha^2 - \frac{\sigma^2 c^2}{(1-s_{AC}^*)^2} \right)$ |
| CS_{Ak}^* | $\frac{n\beta^2}{2\sigma(\beta+\sigma)^2} \left(\alpha - \sigma \frac{c}{1-s_{AN}^*} \right)^2$ | $\frac{n}{8\sigma} \left(\alpha - \sigma \frac{c}{1-s_{AC}^*} \right)^2$ |
| SW_{Ak}^* | $CS_{AN}^* + \Pi_{AN}^* + n \cdot \pi_{AN}^*$ | $CS_{AC}^* + \Pi_{AC}^* + n \cdot \pi_{AC}^*$ |

with s_{AN}^* , that is, $p_{AN}^* = p_{AN}(s_{AN}^*)$, $q_{AN}^* = q_{AN}(s_{AN}^*)$, $\pi_{AN}^* = \pi_{AN}(s_{AN}^*)$, and $\Pi_{AN}^* = \Pi_{AN}(s_{AN}^*)$, which are presented in Table 2.¹⁰

Next, we derive the collusive stage-game equilibrium. Consider collusion among manufacturers in the second round, where the manufacturers seek to maximize their joint profit for any given revenue-sharing rule s set by the platform. Let $\pi^C = \sum_i \pi_i = \sum_i \{(1-s)p_i - c\}q_i$. Solving $\partial \pi^C / \partial p_i = 0$ for all i implies that $p_{AC}(s) = \left(\alpha + \sigma \frac{c}{1-s} \right) / (2\sigma)$. The corresponding quantities, profit of manufacturers, and profit of platform P respectively denote given as shown below.

$$q_{AC}(s) = \frac{\alpha - \sigma \frac{c}{1-s}}{2}, \quad (9)$$

$$\pi_{AC}(s) = (1-s) \frac{\left(\alpha - \sigma \frac{c}{1-s} \right)^2}{4\sigma} \quad (10)$$

$$\Pi_{AC}(s) = \sum_i \left(s \cdot \frac{\alpha + \sigma \frac{c}{1-s}}{2\sigma} \cdot \frac{\alpha - \sigma \frac{c}{1-s}}{2} \right) \quad (11)$$

In the first round, the platform sets the revenue-sharing rule s_{AC}^* which satisfies the following

$$\xi'(s) = -3\alpha^2(1-s)^2 - \{(\beta - \sigma)\alpha c + \beta\sigma c^2\} < 0.$$

The first two inequalities ensure the existence of the solution in $(0, 1 - \sigma c / \alpha)$. The last inequality ensures uniqueness of the solution which satisfies the second-order condition.

¹⁰The fact that $s_{AN} \in (0, 1 - \sigma c / \alpha)$, which we have already shown in footnote 9, ensures all outcomes to be positive (i.e., $p_{AN} > 0$, $q_{AN} > 0$, $\pi_{AN} > 0$, and $\Pi_{AN} > 0$).

first-order condition.¹¹

$$\begin{aligned}\frac{\partial \Pi_{AC}(s_{AC}^*)}{\partial s} &= \frac{n}{4\sigma} \cdot \left[\left(\alpha^2 - \frac{\sigma^2 c^2}{(1 - s_{AC}^*)^2} \right) - s_{AC}^* \frac{2\sigma^2 c^2}{(1 - s_{AC}^*)^3} \right] = 0 \\ &\iff \frac{n}{4\sigma} \cdot \left\{ \alpha^2 - \frac{(1 + s_{AC}^*)\sigma^2 c^2}{(1 - s_{AC}^*)^3} \right\} = 0\end{aligned}\quad (12)$$

The collusive outcomes of the stage game are written with s_{AC}^* , i.e., $p_{AC}^* = p_{AC}(s_{AC}^*)$, $q_{AC}^* = q_{AC}(s_{AC}^*)$, $\pi_{AC}^* = \pi_{AC}(s_{AC}^*)$, and $\Pi_{AC}^* = \Pi_{AC}(s_{AC}^*)$, which are presented in Table 2.¹²

By comparing the stage-game Nash equilibrium with the collusive one under the agency contract, we derive the following lemma.

Lemma 2. *Consider the case with a monopoly platform. Under the agency contract, upstream collusion decreases the revenue-sharing rule (i.e., $s_{AN}^* > s_{AC}^*$), but it increases the retail price (i.e., $p_{AN}^* < p_{AC}^*$), which engenders lower demand, consumer surplus, and social welfare (i.e., $q_{AN}^* > q_{AC}^*$, $CS_{AN}^* > CS_{AC}^*$, and $SW_{AN}^* > SW_{AC}^*$). By colluding, the manufacturers receive greater profit (i.e., $\pi_{AN}^* < \pi_{AC}^*$), whereas the profit of the platform is lower in the presence of the manufacturers' collusion (i.e., $\Pi_{AN}^* > \Pi_{AC}^*$).*

Proof. See the Appendix.

The monopoly platform's optimal revenue-sharing rule is lower in the presence of the manufacturers' collusion. This is true because upstream collusion exacerbates the double marginalization problem, making it unattractive for the platform to impose a high revenue-sharing rule. Lemma 2 also shows that, even though upstream collusion induces the platform to set the lower revenue-sharing rule, it results in the higher retail price eventually.

Upstream collusion enables manufacturers to win the favorable revenue-share and to charge the higher retail price. Therefore, they can gain the greater profit in the collusive equilibrium. By contrast, the platform loses its profit and therefore has no incentive to foster the collusion.

Finally, as in the wholesale contract, upstream collusion shrinks consumer demand. It therefore degrades consumer surplus and social welfare.

Comparison of collusive outcomes under the two contracts Comparing the collusive outcome under the wholesale contract with the one under the agency contract, we have the following lemma.

¹¹ s_{AC}^* is unique over $(0, 1 - \sigma c/\alpha)$ and satisfies the second-order condition. We obtain

$$\begin{aligned}\frac{\partial \Pi_A^C}{\partial s}(0) &= \frac{n}{4\sigma} (\alpha + \sigma c)(\alpha - \sigma c) > 0, \\ \frac{\partial \Pi_A^C}{\partial s} \left(1 - \frac{\sigma c}{\alpha} \right) &= \frac{2n\alpha^2}{\sigma c} (\alpha - \sigma c) < 0, \\ \frac{\partial^2 \Pi_A^C}{\partial s^2}(s) &< 0, \quad \forall s \in \left(0, 1 - \frac{\sigma c}{\alpha} \right).\end{aligned}$$

Therefore, s_{AC}^* is uniquely decided over $(0, 1 - \sigma c/\alpha)$ and satisfies the second-order condition.

¹²The fact that $s_{AC}^* \in (0, 1 - \sigma c/\alpha)$, which we have already shown in footnote 11, ensures that all outcomes are positive (i.e., $p_{AC}^* > 0$, $q_{AC}^* > 0$, $\pi_{AC}^* > 0$, and $\Pi_{AC}^* > 0$).

Lemma 3. *Consider the case with a monopoly platform. Presume also that the upstream manufacturers collude to maximize their joint profit. Compared to the wholesale contract, the retail price is lower under the agency contract (i.e., $p_{WC}^* > p_{AC}^*$), which engenders greater demand (i.e., $q_{WC}^* < q_{AC}^*$). Then, consumer surplus and social welfare are higher under the agency contract (i.e., $CS_{WC}^* < CS_{AC}^*$ and $SW_{WC}^* < SW_{AC}^*$). The manufacturers receive greater profit under the wholesale contract (i.e., $\pi_{WC}^* > \pi_{AC}^*$), although the platforms prefer the agency contract (i.e., $\Pi_{WC}^* < \Pi_{AC}^*$).*

Proof. See the Appendix.

When the manufacturers collude to maximize their joint profit, the retail price is lower under the agency contract than under the wholesale contract, which generates the greater demand and then improves both consumer surplus and social welfare. In this respect, the agency contract is not necessarily anticompetitive, but rather improves social welfare.

Furthermore, the manufacturers prefer the wholesale contract to the agency contract, whereas the platform has the opposite preference. In other words, every firm prefers a contract that guarantees itself the first move. This result is also presented by Johnson (2017), although upstream collusion is not considered in his model. Consequently, the presence of upstream collusion does not alter the preference of each firm related to the contract type.

2.2 Infinitely repeated game

This subsection presents examination of an infinitely repeated game in which the monopoly platform and manufacturers play the above stage-game over period ($t = 1, 2, \dots, \infty$). The game is of common knowledge and perfect monitoring. Let $\delta \in (0, 1)$ be a common discount factor. The payoff of each player is given as the sum of the discounted stage-game payoff. They maximize their expected payoff. As a solution concept, we use a Markov-perfect equilibrium with a class of strategies described later for each contract.

The analyses presented in this paper include the assumption that manufacturers sustain their joint profit maximizing collusion through infinite Nash-reversion, where deviation by one manufacturer from the collusion is punished by playing the Nash equilibrium strategy of the stage-game forever. We determine the value of the critical discount factor, which is the lowest discount factor with which manufacturers' joint profit maximizing collusion can be sustained.¹³ We stipulate that the agency contract *facilitates* upstream collusion if it *reduces* the critical discount factor.

Critical discount factor under the wholesale contract Here we derive the critical discount factor obtained under the wholesale contract, denoted as δ_W^* . Under the wholesale contract, manufacturers collude to maximize their joint profit before the platform moves. This is the standard

¹³An alternative means exists of finding the collusive action that maximizes the manufacturers' joint profit given the discount factor and punishment strategy. However, following the existing studies (e.g., Davidson (1984), more recently, Nocke and White (2007) and Reisinger and Thomes (2017)), we consider the maximization of joint profit in the stage-game as a collusive action taken by manufacturers.

case of the literature on collusion in the vertically related market. We consider the following Nash-reversion trigger strategies for all players.

- Strategy of manufacturers: Manufacturers have the following two phases depending on the history they face.
 - Collusion phase: For any history where no manufacturer has deviated, all manufacturers take a symmetric action that maximizes their joint stage-game profit given $p_i(w_i) = \alpha/(2\sigma) + w_i/2$ for any $i = 1, \dots, n$.
 - Punishment phase: For any history where at least one manufacturer has already deviated from the joint-profit maximization, all manufacturers play the stage-game Nash equilibrium strategy.
- Strategy of monopoly platform: Given a pair of wholesale prices set by manufacturers, the monopoly platform chooses retail prices p_i for all i .

We have already derived the stage-game Nash equilibrium and the collusive equilibrium in Section 2.1. Consequently, here, we compute the deviation payoff.

As in the static game, the best response of platform P at the second round is $p_i(w_i) = \alpha/(2\sigma) + w_i/2$. We consider the deviation by manufacturer 1 in the first round, with no loss of generality. Consequently, the other manufacturers set the collusive wholesale price (i.e., $w_j = w_{WC}^*$ for $j = 2, \dots, n$). Then, the profit of manufacturer 1 can be written as

$$\begin{aligned} \pi_1^D(w_1) &= (w_1 - c) \left(\alpha - \beta p_1 + \gamma \sum_{j \neq 1} p_j \right) \\ &= (w_1 - c) \left[\alpha - \beta \left(\frac{\alpha}{2\sigma} + \frac{w_1}{2} \right) + (\beta - \sigma) \left(\frac{\alpha}{2\sigma} + \frac{w_{WC}^*}{2} \right) \right]. \end{aligned} \quad (13)$$

Solving $\partial \pi_1^D(w_1)/\partial w_1 = 0$ yields $w_{WD}^* = [\alpha(\beta + \sigma) + \sigma\{2\beta + (\beta - \sigma)\}c]/(4\beta\sigma)$. We use subscript D to denote the *deviation* equilibrium. The deviation profit can be computed as $\pi_{WD}^* = (\beta + \sigma)^2(\alpha - \sigma c)^2/(32\beta\sigma^2)$.

Then collusion among manufacturers is sustainable if and only if the following inequality holds.

$$\frac{\pi_{WC}^*}{1 - \delta} \geq \pi_{WD}^* + \frac{\delta \pi_{WN}^*}{1 - \delta} \quad (14)$$

By solving $\frac{\pi_{WC}^*}{1 - \delta} = \pi_{WD}^* + \frac{\delta \pi_{WN}^*}{1 - \delta}$, we derive the critical discount factor under the wholesale contract, δ_W^* .

Proposition 1. *Consider the case with a monopoly platform. Under the wholesale contract, collusion among manufacturers is sustainable if and only if the discount factor is sufficiently high to satisfy*

$$\delta \geq \delta_W^* \equiv \frac{\beta^2 + \sigma^2 + 2\beta\sigma}{\beta^2 + \sigma^2 + 6\beta\sigma}. \quad (15)$$

Proof. It can be shown with simple calculations. □

One can readily show that the critical discount factor δ_W^* increases with n and γ and that it decreases in β , i.e., $\partial\delta_W^*/\partial n > 0$, $\partial\delta_W^*/\partial\gamma > 0$, and $\partial\delta_W^*/\partial\beta < 0$. The tougher competition (e.g., a larger number of manufacturers and/or a higher degree of substitution) engenders the higher δ_W^* , which is relevant to a well-known result that the collusion becomes more difficult to sustain as the competition gets tougher.¹⁴

Critical discount factor under the agency contract Next, we derive the critical discount factor under the agency contract, denoted as δ_A^* . Unlike the wholesale contract, under the agency contract, one player moves before the formation of collusion. The platform can design its action strategically to influence collusion among manufacturers. The platform might have an incentive to hinder or foster the collusion by manipulating its action. Therefore, we must consider the equilibrium path by which the platform deters manufacturers' joint profit maximization. To this end, we consider the following Nash-reversion trigger strategies.

- Strategy of manufacturers: Manufacturers have the following two phases depending on the history they face.
 - Collusion phase: For any history where no manufacturer has deviated, all manufacturers take a symmetric action that maximizes their joint profit if the platform sets $s \in \mathcal{S}$. Otherwise, if the platform sets $s \notin \mathcal{S}$, then the manufacturers follow the stage-game Nash equilibrium strategy.
 - Punishment phase: For any history where at least one manufacturer has already deviated despite the platform set $s \in \mathcal{S}$ in a collusion phase, all manufacturers play the stage-game Nash equilibrium strategy for any given s .
- Strategy of monopoly platform: The monopoly platform chooses a revenue-sharing rule s at every period.

As we have explained, we assume that the platform takes a Markov strategy, which means that the action of the platform depends only on the phase of manufacturers. This assumption is made to keep the platform from actively engaging in collusion in the sense that it uses no spontaneous punishment strategy to manipulate the actions of the other players.

We identify the value of the critical discount factor under the strategies specified above. To do so, we specifically examine the platform-preferred pure strategy equilibria with maximal \mathcal{S} : the set \mathcal{S} is maximal given the strategy of the platform; and the equilibria are chosen to maximize the equilibrium payoff of the platform. This selection criterion has two implications. First, the maximal \mathcal{S} means that the manufacturers engage in collusion as long as possible. Second, the

¹⁴The opposite result is obtained in the common retailer case of Reisinger and Thomes (2017). That is, they demonstrated that the critical discount factor decreases in γ when each manufacturer offers a two-part tariff contract to the common retailer.

platform-preferred equilibria reflect the incentive of the platform on the stability of collusion among manufacturers. We proceed to measure the stability of upstream collusion while considering these two implications.

We derive the deviation payoff given a revenue-sharing rule s because we have already obtained the stage-game Nash equilibrium and the collusive equilibrium in Section 2.1. Without loss of generality, we consider the deviation by manufacturer 1 in the second round, i.e., the other manufacturers charge the collusion price for a given s (i.e., $p_j = p_{AC}(s)$ for $j = 2, \dots, n$). Then, the profit of manufacturer 1 can be expressed as presented below.

$$\begin{aligned}\pi_1^D(p_1) &= \{(1-s)p_1 - c\} \left(\alpha - \beta p_1 + \gamma \sum_{j \neq 1} p_j \right) \\ &= \{(1-s)p_1 - c\} \{\alpha - \beta p_1 + (n-1)\gamma p_{AC}(s)\}\end{aligned}\quad (16)$$

Solving $\partial \pi_1^D(p_1) / \partial p_1 = 0$ yields $p_{AD}(s) = \left\{ \alpha + (n-1)\gamma p_C + \beta \frac{c}{1-s} \right\} / (2\beta)$. The deviation profit of manufacturer 1 is computed as $\pi_{AD}(s) = (1-s)(\beta + \sigma)^2 \left(\alpha - \sigma \frac{c}{1-s} \right)^2 / (16\beta\sigma^2)$.

Let s^* be the equilibrium revenue-sharing rule that the platform sets in the collusion phase. Because the platform follows Markov strategy, the platform chooses s^* as long as begin in the collusion phase. Then, the collusion among manufacturers is sustainable if and only if the following inequality holds:

$$\pi_{AD}(s) - \pi_{AC}(s) \leq \frac{\delta}{1-\delta} \{ \pi_{AC}(s^*) - \pi_{AN}(s_{AN}^*) \}.\quad (17)$$

We use $\tilde{s}(\delta, s^*)$ to denote the value of revenue-sharing rule s such that condition (17) holds with equality. Given a pair of parameters $(\alpha, \beta, \sigma, c)$, one can interpret \tilde{s} as a function of discount factor δ and the equilibrium revenue-sharing rule s^* . Let $\delta_A(s, s^*)$ be the value of δ such that condition (17) holds with equality for any given s and the equilibrium revenue-sharing rule s^* . That is, $\delta_A(s, s^*)$ is an inverse function of \tilde{s} with respect to δ at a given s^* . Then, one can infer that condition (17) holds if and only if $\delta \geq \delta_A(s, s^*)$. We have the following lemma on the property of $\delta_A(\cdot, \cdot)$.

Lemma 4. *The following statements hold;*

- (i) $\delta_A(s, s^*)$ is increasing in s^* and decreasing in s .
- (ii) $\tilde{\delta}_A(s) := \delta_A(s, s)$ is increasing in s .
- (iii) $\tilde{\delta}_A(s_{AN}^*) = \delta_W^*$.

Proof. See the Appendix. □

Part (i) of Lemma 4 first shows that the collusion among manufacturers is harder to sustain as the platform charges a higher equilibrium revenue-sharing rule. Additionally, it implies that upstream collusion can be sustained more easily if the platform charges a revenue-sharing rule higher than the equilibrium level in a period. The intuition is that an increase in the future

revenue-sharing rule s^* decreases the future value of sustaining collusion, whereas an increase in the current revenue-sharing rule decreases the current net value of deviating from the collusion.

This result also implies that collusion among manufacturers is sustainable if and only if $s \geq \tilde{s}(\delta, s^*)$. In other words, $\tilde{s}(\delta, s^*)$ is the smallest revenue-sharing rule at which the upstream collusion can be sustained.

Similarly, as shown in Lemma 4 (ii), there exists threshold $\tilde{s}_A(\delta)$ below which the collusion among manufacturers can be sustainable. That is, if $s^* \leq \tilde{s}_A(\delta)$, then the manufacturers can sustain their joint profit maximizing collusion. This result generates tension between fostering and hindering collusion in the platform's decision on s^* . Setting a high revenue-sharing rule for subsequent future periods hinders the formation of upstream collusion overall, but in each period, the platform should set a low revenue-sharing rule to hinder collusion.

In sum, given the equilibrium revenue-sharing rule s^* , a maximal set $\mathcal{S}^*(s^*)$ is determined as

$$\mathcal{S}^*(s^*) = \{s \in [0, 1] \mid s > \tilde{s}(\delta, s^*)\}.. \quad (18)$$

Furthermore, given the maximal \mathcal{S}^* , the platform determines the equilibrium revenue-sharing rule s^* to satisfy

$$s^* = \arg \max_s \Pi_A(s, \mathcal{S}^*), \quad (19)$$

where

$$\Pi_A(s, \mathcal{S}^*) = \begin{cases} \Pi_{AC}(s) & \text{if } s \in \mathcal{S}^* \\ \Pi_{AN}(s) & \text{if } s \notin \mathcal{S}^* \end{cases} \quad (20)$$

Finally, a pair of the revenue-sharing rule and the maximal set $(s^*, \mathcal{S}^*(s^*))$ characterizes the equilibrium under consideration. We obtain the following proposition on the equilibrium revenue-sharing rule s^* and the critical discount factor δ_A^* under the agency contract.

Proposition 2. *Consider the case with a monopoly platform. In the platform-preferred equilibrium under the agency contract, the platform sets the revenue-sharing rule s^* as*

$$s^* = \begin{cases} s_{AN}^* & \text{if } \delta < \delta_A^* , \\ s_{AC}^* & \text{if } \delta \geq \delta_A^* , \end{cases} \quad (21)$$

where $\delta_A^* = \tilde{\delta}_A(s_{AN}^*)$. Therefore, the collusion among manufacturers is sustained if and only if $\delta \geq \delta_A^*$.

Proof. See the Appendix.

As shown in Lemma 2, it always holds that $\Pi_{AN}^* > \Pi_{AC}^*$. Consequently, it would be best for the platform to choose $s = s_{AN}^*$ as long as it deters collusion among the manufacturers. That is, if the joint profit maximization among manufacturers is not sustainable at s_{AN}^* , then the platform chooses $s = s_{AN}^*$. From Lemma 4 (i), threshold $\tilde{s}(\delta, s^*)$ is decreasing in discount factor δ . For a

sufficiently small δ , threshold $\tilde{s}(\delta, s^*)$ tends to be greater than s_{AN}^* . Then, charging $s = s_{AN}^*$ at every period is optimal for the platform because it can deter the manufacturers' collusion effectively.

As δ becomes larger, threshold $\tilde{s}(\delta, s^*)$ becomes smaller. Once δ exceeds $\tilde{\delta}_A(s_{AN}^*)$, the threshold $\tilde{s}_A(\delta)$ falls below s_{AN}^* . Then, the platform cannot deter the collusion merely by choosing $s = s_{AN}^*$. To deter collusion, it is necessary to charge $s = \tilde{s}(\delta, s^*) < s_{AN}^*$. However, this strategy cannot be supported as a pure strategy equilibrium for the following reason. Presuming that the platform sets $s = \tilde{s}(\delta, s^*)$ at all subsequent future periods, then the platform can derive greater profit by setting $s = s_{AN}^*$ for each period.

Alternatively, for $\delta \geq \tilde{\delta}_A(s_{AN}^*)$, it is more profitable for the platform to accommodate the upstream collusion by choosing $s = s_{AC}^*$, which is the unique pure strategy equilibrium. In other words, it is not profitable for the platform to deviate to other s to deter upstream collusion. The platform must set $s = \tilde{s}(\delta, s_{AC}^*)$ to deter the collusion, which is less than s_{AC}^* because $s_{AN}^* > s_{AC}^*$ (Lemma 2) and $\tilde{\delta}_A$ is increasing (Lemma 4 (ii)). Such a deviation is not profitable. Therefore, the platform sets $s = s_{AC}^*$ in equilibrium.

Comparison of the two critical discount factors We compare the critical discount factor under the wholesale contract with that obtained under the agency contract. Results derived in Propositions 1 and 2 suggest the following proposition.

Proposition 3. *In the model with a monopoly platform, the agency contract neither facilitates nor obstructs upstream collusion. Formally, $\delta_W^* = \delta_A^*$ holds.*

Proof. This follows from $\delta_A^* = \tilde{\delta}_A(s_{AN}^*)$ in Proposition 2 and $\tilde{\delta}_A(s_{AN}^*) = \delta_W^*$ in Lemma 4 (iii). \square

Proposition 3 shows that, in the monopoly platform case, whether upstream collusion arises is independent of the type of contract. Furthermore, Lemma 3 shows that the agency contract provides better consumer surplus and better social welfare than the wholesale contract when the manufacturers collude. In this respect, the agency contract cannot be characterized as anticompetitive in the monopoly platform case. As presented in Section 4.1, we discuss the robustness of the results obtained in this section.

3 Platform Competition

The discussion in this section addresses how the presence of platform competition affects the stability of upstream collusion. Section 2 has shown that, if the market has only one platform, the agency contract neither facilitates nor obstructs collusion among manufacturers.

By contrast, with platform competition, no platform alone can simply control the formation of manufacturers' collusion because strategies of the other platforms also affect the formation of the collusion. Additionally, competing platforms, by keeping in step mutually for making the upstream collusion sustainable, can avoid fierce competition among themselves with respect to the revenue-sharing rules. In other words, accepting upstream collusion would ease the platform competition. These effects can be expected to make the platforms favor the manufacturers' collusion, which is

in sharp contrast with the finding of the monopoly platform case. To illustrate and confirm this conjecture, we use a bilateral duopoly model, as used in Dobson and Waterson (1996).

3.1 The stage-game

Herein, after we describe only those components of the stage-game that differ from those of the monopoly platform case, we analyze static equilibrium and collusion among manufacturers under the two contracts. Subsequently, we compare those stage-game outcomes.

Setting of the stage-game We consider two upstream manufacturers ($u = 1, 2$) and two downstream platforms ($d = 1, 2$). Both manufacturers produce differentiated goods with a constant marginal cost, which is normalized to zero. Actually, all of our results hold even when we introduce a small marginal cost $c > 0$ by the continuity of the model. Let p_d^u and q_d^u be the price and the quantities of manufacturer u 's good demanded at platform d .

Following Dobson and Waterson (1996), we assume a representative consumer with the following utility function:¹⁵

$$U = \sum_{u,d} q_d^u - \sum_{u,d} \frac{1}{2} (q_d^u)^2 - \lambda (q_d^u q_{-d}^u + q_d^{-u} q_{-d}^{-u}) - \mu (q_d^u q_d^{-u} + q_{-d}^u q_{-d}^{-u}) - \lambda \mu (q_d^u q_{-d}^{-u} + q_{-d}^u q_d^{-u}) \quad \text{for } u, d = 1, 2, \quad (22)$$

where $\mu \in (0, 1)$ and $\lambda \in (0, 1)$ respectively represent the degrees of substitution between the upstream manufacturers and between the downstream platforms.

Utility-maximization subject to the budget constraint yields the inverse demand functions: $p_d^u = 1 - (q_d^u + \lambda q_{-d}^u) - \mu (q_d^{-u} + \lambda q_{-d}^{-u})$ for $u, d = 1, 2$. Solving for quantities, we can derive the demand function as follows.

$$q_d^u = \frac{(1 - \lambda)(1 - \mu) - p_d^u + \mu p_d^{-u} + \lambda (p_{-d}^u - \mu p_{-d}^{-u})}{(1 - \lambda^2)(1 - \mu^2)} \quad \text{for } u, d = 1, 2 \quad (23)$$

For simplicity, we assume that $\lambda \geq \mu/(16 - 9\mu)$.¹⁶

Stage-game analysis under the wholesale contract Under the wholesale contract, manufacturer u sets wholesale prices (w_1^u, w_2^u) in the first round; then platform d sets retail prices (p_d^1, p_d^2) in the second round.

First, we derive the stage-game Nash equilibrium. In the second round, solving the platforms' profit maximization problems, we obtain the second-stage price $p_d^u(w)$. Using this, the corresponding second-round quantities are given as

$$q_d^u(w) = \frac{\lambda (w_d^u - \mu w_d^{-u}) - (2 - \lambda^2)(w_d^u - \mu w_d^{-u})}{(4 - \lambda^2)(1 - \lambda^2)(1 - \mu^2)} + \frac{1}{(2 - \lambda)(1 + \lambda)(1 + \mu)}. \quad (24)$$

¹⁵This utility function and the resulting demand function have been used widely in the literature (e.g., Johansen and Vergé, 2017; Foros et al., 2017; Lu, 2017).

¹⁶ $\lambda \geq 1/7$ is a sufficient condition for this assumption. Therefore, this assumption is not too restrictive.

Table 3: Stage-game equilibrium and collusive equilibrium under the wholesale contract with platform competition

| $k = \{N, C\}$ | Stage-game Nash equilibrium (N) | Collusive equilibrium (C) |
|-----------------|---|--|
| w_{Wk}^{**} | $\frac{1-\mu}{2-\mu}$ | $\frac{1}{2}$ |
| p_{Wk}^{**} | $\frac{(1-\lambda)(2-\mu)+1-\mu}{(2-\lambda)(2-\mu)}$ | $\frac{3-2\lambda}{2(2-\lambda)}$ |
| q_{Wk}^{**} | $\frac{1}{(1+\lambda)(2-\lambda)(1+\mu)(2-\mu)}$ | $\frac{1}{2(2-\lambda)(1+\lambda)(1+\mu)}$ |
| π_{Wk}^{**} | $\frac{2(1-\mu)}{(1+\lambda)(1+\mu)(2-\lambda)(2-\mu)^2}$ | $\frac{1}{2(2-\lambda)(1+\lambda)(1+\mu)}$ |
| Π_{Wk}^{**} | $\frac{2(1-\lambda)}{(1+\lambda)(1+\mu)(2-\lambda)^2(2-\mu)^2}$ | $\frac{3-2\lambda}{2(2-\lambda)^2(1+\lambda)(1+\mu)}$ |
| CS_{Wk}^{**} | $\frac{2}{(1+\lambda)(2-\lambda)^2(1+\mu)(2-\mu)^2}$ | $\frac{1}{2(1+\lambda)(2-\lambda)^2(1+\mu)}$ |
| SW_{Wk}^{**} | $\frac{2+4(1-\mu)(2-\lambda)+4(1-\lambda)}{(1+\lambda)(2-\lambda)^2(1+\mu)(2-\mu)^2}$ | $\frac{11-6\lambda}{2(1+\lambda)(2-\lambda)^2(1+\mu)}$ |

In the first round, the stage-game equilibrium can be derived by solving the following problem for each manufacturer: $\max_{(w_d^u)_{d=1,2}} \sum_{d'} (w_{d'}^u - c) q_{d'}^u(w)$, as analyzed by Lu (2017). The outcomes of the stage-game Nash equilibrium are presented in Table 3. We use superscript ‘**’ to denote the equilibrium in the model with the two competing platforms.

Second, we consider joint profit maximization by manufacturers, which can be derived by solving the following problem: $\max_{(w_d^u)_{u,d=1,2}} \sum_u \sum_{d'} (w_{d'}^u - c) q_{d'}^u(w)$. The resulting outcomes of the collusive equilibrium are also presented in Table 3.

By comparing the stage-game Nash equilibrium with the collusive one under the wholesale contract, we derive the following lemma.

Lemma 5. *One can consider a case with two competing platforms. Under the wholesale contract, upstream collusion increases wholesale and retail prices (i.e., $w_{WN}^{**} < w_{WC}^{**}$ and $p_{WN}^{**} < p_{WC}^{**}$), which decrease the quantities demanded, consumer surplus, and social welfare (i.e., $q_{WN}^{**} > q_{WC}^{**}$, $CS_{WN}^{**} > CS_{WC}^{**}$, and $SW_{WN}^{**} > SW_{WC}^{**}$). The manufacturers receive greater profit (i.e., $\pi_{WN}^{**} < \pi_{WC}^{**}$), whereas the profit of platforms declines (i.e., $\Pi_{WN}^{**} > \Pi_{WC}^{**}$).*

Proof. It can be shown with simple calculations. □

Lemma 5 presents the same result as that of Lemma 1. Therefore, under the wholesale contract, the effects of upstream collusion are irrelevant to the existence of platform competition. As in the monopoly platform case, the upstream collusion is profitable only to the manufacturers; it is deleterious to the platforms, consumer surplus, and social welfare.

Stage-game analysis under the agency contract Under the agency contract, platform d sets a revenue-sharing rule s_d in the first round; then manufacturer u sets retail prices (p_1^u, p_2^u) in the second round.

We first derive the stage-game Nash equilibrium. In the second round, given (s_1, s_2) , each

Table 4: Stage-game equilibrium and collusive equilibrium under the agency contract with platform competition

| $k = \{N, C\}$ | Stage-game Nash equilibrium (N) | Collusive equilibrium (C) |
|-----------------|--|---|
| s_{Ak}^{**} | $\frac{(2-\mu)(1-\lambda)^2}{2-\mu(1+\lambda)}$ | $1 - \lambda^2$ |
| p_{Ak}^{**} | $\frac{1-\mu}{2-\mu}$ | $\frac{1}{2}$ |
| q_{Ak}^{**} | $\frac{1}{(1+\lambda)(1+\mu)(2-\mu)}$ | $\frac{1}{2(1+\lambda)(1+\mu)}$ |
| π_{Ak}^{**} | $\frac{2\lambda(1-\mu)\{2\lambda-\mu(1+\lambda)\}}{(1+\lambda)(1+\mu)(2-\mu)^2\{2-\mu(1+\lambda)\}}$ | $\frac{\lambda^2}{2(1+\lambda)(1+\mu)}$ |
| Π_{Ak}^{**} | $\frac{2(1-\lambda)(1-\mu)}{(1+\mu)(2-\mu)\{2-\mu(1+\lambda)\}}$ | $\frac{1-\lambda^2}{2(1+\lambda)(1+\mu)}$ |
| CS_{Ak}^{**} | $\frac{2}{(1+\lambda)(1+\mu)(2-\mu)^2}$ | $\frac{1}{2(1+\lambda)(1+\mu)}$ |
| SW_{Ak}^{**} | $\frac{6-4\mu}{(1+\lambda)(1+\mu)(2-\mu)^2}$ | $\frac{3}{2(1+\lambda)(1+\mu)}$ |

manufacturer u solves the following problem:

$$\max_{p_1^u, p_2^u} [(1-s_1)p_1^u - c]q_1^u + [(1-s_2)p_2^u - c]q_2^u. \quad (25)$$

Solving the maximization problem above, we derive the retail prices, which are denoted as $p_d^N(s_1, s_2)$ for $d = 1, 2$. Let $q_d^N(s_1, s_2)$ be the corresponding quantities.

In the first round, each platform chooses revenue-sharing rule s_d to maximize $\Pi_{dA}^N(s_d, s_{-d}) = 2s_d p_d^N(s_d, s_{-d}) q_d^N(s_d, s_{-d})$. By solving this problem, we obtain a symmetric Nash equilibrium s_{AN}^{**} . This stage-game Nash equilibrium has been analyzed by Foros et al. (2017) and Lu (2017), as presented in Table 4.

Second, we consider joint profit maximization by manufacturers. In this case, given (s_1, s_2) , each manufacturer u solves the following problem:

$$\max_{(p_d^u)_{u,d=1,2}} \sum_{u=1,2} ([(1-s_1)p_1^u - c]q_1^u + [(1-s_2)p_2^u - c]q_2^u). \quad (26)$$

Solving the maximization problem above, we derive the retail prices, which are denoted as $p_d^C(s_d, s_{-d})$ for $d = 1, 2$. Let $q_d^C(s_d, s_{-d})$ be the resulting quantities.

In the first round, platform d solves the maximization problem: $\max_{s_d} \sum_{u=1,2} s_d p_d^C(s_d, s_{-d}) q_d^C(s_d, s_{-d})$. The first-order condition is given as

$$\sum_{u=1,2} \left[p_d^C q_d^C + s_d \left[\frac{\partial p_d^C}{\partial s_d} \left(q_d^C + \frac{\partial q_d^C}{\partial p_d} p_d^C \right) + \frac{\partial p_{-d}^C}{\partial s_d} \frac{\partial q_d^C}{\partial p_{-d}} p_d^C \right] \right] = 0. \quad (27)$$

Invoking symmetry, $s_d = s_{-d} = s$, we obtain $s_{AC}^{**} = 1 - \lambda^2 > s_{AN}^{**}$. The other resulting outcomes are also presented in Table 4.

By comparing the stage-game Nash equilibrium with the collusive one under the agency contract,

we derive the following lemma.

Lemma 6. *Consider the case with two competing platforms. Under the agency contract, the revenue-sharing rule and the retail price are higher in the presence of the manufacturers' collusion (i.e., $s_{AN}^{**} < s_{AC}^{**}$ and $p_{AN}^{**} < p_{AC}^{**}$), which decrease the quantities demanded, consumer surplus, and social welfare (i.e., $q_{AN}^{**} > q_{AC}^{**}$, $CS_{AN}^{**} > CS_{AC}^{**}$, and $SW_{AN}^{**} > SW_{AC}^{**}$). Both the manufacturers and the platforms receive greater profits (i.e., $\pi_{AN}^{**} < \pi_{AC}^{**}$ and $\Pi_{AN}^{**} < \Pi_{AC}^{**}$).*

Proof. The result can be shown using simple calculations. □

The existence of platform competition brings about two changes. The first is that the revenue-sharing rule is higher in the presence of upstream collusion ($s_{AN}^{**} < s_{AC}^{**}$). In the monopoly platform case, we showed that upstream collusion induces the platform to set the lower revenue-sharing rule because of the double marginalization problem. In this respect, the opposite result is obtained with platform competition.

Without upstream collusion, the platforms face fierce competition with respect to their revenue-sharing rules. However, by keeping mutually in step for making the upstream collusion sustainable, they can mitigate the competition. As a result, not only the manufacturers but also the platforms can receive greater profit from the upstream collusion, which is the second change. That result implies that the platforms would have an incentive to accept collusion also in their repeated interactions.

Comparison of collusive outcomes under the two contracts Here, we compare the collusive outcome under the wholesale contract with the outcome obtained under the agency contract.

Lemma 7. *One can consider a case with two competing platforms. Presume that the upstream manufacturers collude to maximize their joint profit. Compared to the wholesale contract, the retail price is lower under the agency contract (i.e., $p_{WC}^{**} > p_{AC}^{**}$), which engenders greater demand, consumer surplus, and social welfare (i.e., $q_{WC}^{**} < q_{AC}^{**}$, $CS_{WC}^{**} < CS_{AC}^{**}$, and $SW_{WC}^{**} < SW_{AC}^{**}$). The manufacturers receive greater profit under the wholesale contract (i.e., $\pi_{WC}^{**} > \pi_{AC}^{**}$), although the platforms prefer the agency contract (i.e., $\Pi_{WC}^{**} < \Pi_{AC}^{**}$).*

Proof. The result can be shown using simple calculations. □

Lemma 7 shows qualitatively the same result as that obtained in Lemma 3. Therefore, when manufacturers collude to maximize their joint profit, the existence of platform competition does not affect the comparison between the two contracts.

3.2 Infinitely repeated game

This subsection presents examination of an infinitely repeated game in which two platforms and two manufacturers play the above stage-game over period ($t = 1, 2, \dots, \infty$). As in the case of monopoly platform studied in Section 2, the game is of common knowledge and perfect monitoring with a common discount factor δ . Furthermore, manufacturers sustain their joint profit maximizing

collusion through infinite Nash-reversion. We derive the values of the critical discount factor for the respective contracts.

Critical discount factor under the wholesale contract We use δ_W^{**} to denote the critical discount factor under the wholesale contract. We consider the following Nash-reversion trigger strategies for all players.

- Strategy of manufacturers: Manufacturers have two phases.
 - Collusion phase: For any history in which no manufacturer has deviated, all manufacturers take a symmetric action that maximizes their joint stage-game profit given $p_d^u(w)$.
 - Punishment phase: For any history in which at least one manufacturer has already deviated from the collusion, all manufacturers play the stage-game Nash equilibrium strategy.
- Strategy of platforms: Given the wholesale prices set by the manufacturers, each platform d chooses retail prices (p_d^1, p_d^2) in every period.

We have already derived the stage-game Nash equilibrium and the collusive equilibrium in Section 3.1. Therefore, we compute the deviation payoff here. Under the wholesale contract, manufacturers move before platforms do. Consequently, the platforms' best response strategies given a pair of wholesale prices are unchanged. Under those circumstances, the first-order condition for the best deviation from the joint profit maximization can be written as

$$\frac{\partial \pi^u}{\partial w_d^u} = q_d^u + w_d^u \frac{\partial q_d^u}{\partial w_d^u} + w_{-d}^u \frac{\partial q_{-d}^u}{\partial w_d^u} = 0 \quad \text{s.t. } w_d^{-u} = w^C \quad \text{for } d = 1, 2. \quad (28)$$

Solving this equation, we obtain the deviation strategy w_{WD}^{**} and the corresponding profit of the manufacturer π_{WD}^{**} , respectively, as

$$w_{WD}^{**} = \frac{2 - \mu}{4}, \quad \pi_{WD}^{**} = \frac{(2 - \mu)^2}{8(2 - \lambda)(1 + \lambda)(1 - \mu^2)}. \quad (29)$$

Consequently, the critical discount factor δ_W^{**} is computed as follows.

$$\delta_W^{**} = \frac{\pi_{WD}^{**} - \pi_{WC}^{**}}{\pi_{WD}^{**} - \pi_{WN}^{**}} = \frac{\frac{(2-\mu)^2}{8(2-\lambda)(1+\lambda)(1-\mu^2)} - \frac{1}{2[(2-\lambda)(1+\lambda)(1+\mu)]}}{\frac{(2-\mu)^2}{8(2-\lambda)(1+\lambda)(1-\mu^2)} - \frac{2(1-\mu)}{(1+\lambda)(1+\mu)(2-\lambda)(2-\mu)^2}} \quad (30)$$

Proposition 4. *In the model with two competing platforms, collusion under the wholesale contract among manufacturers is sustainable if and only if $\delta \geq \delta_W^{**}$.*

Critical discount factor under the agency contract Next, we derive the critical discount factor under the agency contract, denoted as δ_A^{**} . We consider the following Nash-reversion trigger strategies for all players.

- Strategy of manufacturers: Manufacturers have two phases.
 - Collusion phase: For any history in which no manufacturer has deviated, all manufacturers choose a symmetric action that maximizes their joint profit for a given $(s_1, s_2) \in \mathcal{S}_d \subset [0, 1]^2$. Otherwise, given $(s_1, s_2) \notin \mathcal{S}_d$, the manufacturers follow the stage-game Nash equilibrium strategy.
 - Punishment phase: For any history in which at least one manufacturer has already deviated, all manufacturers play the stage-game Nash equilibrium for any given (s_1, s_2) .
- Strategy of platforms: Each platform d chooses revenue-sharing rule s_d at every period.

As in the monopoly platform case, we assume that the platforms take Markov strategies. Additionally, we confine our attention to the platform-preferred symmetric equilibrium with maximal \mathcal{S}_d .

We have already derived the stage-game Nash equilibrium and the collusive equilibrium in Section 3.1. To derive the critical discount factor, we obtain the deviation payoff. Given (s_1, s_2) , the optimal deviation from joint profit maximization is obtained by solving the following problem.

$$\max_{p_1^u, p_2^u} ((1-s_1)p_1^u - c]q_1^u + [(1-s_2)p_2^u - c]q_2^u) \quad s.t. \quad p_d^{-u} = p_d^C(s_1, s_2) \quad (31)$$

Let $p_{AD}(s_1, s_2)$ be the solution to this problem and $\pi_{AD}(s_1, s_2)$ be the corresponding profit.

Consider the symmetric equilibrium with revenue-sharing rule s^{**} in the collusion phase. We assume Nash-reversion punishment. Therefore, joint profit maximization is sustainable if and only if the following inequality holds:

$$\pi_{AD}(s_1, s_2) - \pi_{AC}(s_1, s_2) \leq \frac{\delta}{1-\delta} \{\pi_{AC}(s^{**}, s^{**}) - \pi_{AN}^{**}\}. \quad (32)$$

Let $\delta_A(s_1, s_2, s^{**})$ be the value of δ such that the above condition holds with equality. Then, condition (32) holds if and only if $\delta \geq \delta_A(s_1, s_2, s^{**})$. We have the following characterization on $\delta_A(s_1, s_2, s^{**})$.

Lemma 8. *The following statements hold:*

- (i) $\delta_A(s, s, s)$ is increasing in s .
- (ii) For any s such that $\delta_A(s, s, s^{**}) \in [0, 1]$ and for any $d = 1, 2$, $\delta_A(s_1, s_2, s^{**})$ is increasing in s_d in the neighborhood of $(s_1, s_2, s^{**}) = (s, s, s^{**})$.

Proof. See the Appendix. □

Lemma 8 states that (i) if symmetrically set time-invariant revenue-sharing rules increase simultaneously, then collusion becomes more difficult to sustain, and (ii) an increase in the revenue-sharing rule of one platform from symmetrically set ones also makes the collusion more difficult. The intuition for the second result is that under the platform competition, an increase in the revenue-sharing rule of one platform has another effect: a production substitution effect. These effects change the relative magnitude of the impact of the revenue-sharing rule on the stability of collusion. As a result, the collusion becomes increasingly difficult to sustain as a platform sets a higher revenue-sharing rule.

With this lemma, one can naturally expect that $\delta_A(s_1, s_2)$ is increasing in both arguments in the relevant ranges of (s_1, s_2) such that $s_1 \neq s_2$. However, specifying the conditions under which $\delta_A(s_1, s_2)$ is increasing in both arguments requires tedious calculations, without elucidating any important insights. Therefore, we simply assume that parameter values are such that $\delta_A(s_1, s_2)$ are increasing in s_1 and s_2 in any relevant values of (s_1, s_2) .

Assumption 1. *Parameters λ and μ take values such that $\delta_A(s_1, s_2, s^*)$ is increasing in s_1 and s_2 for all $(s_1, s_2) \in [s_{AN}^{**}, s_{AC}^{**}]^2$.*

Finally, one can consider the platforms' incentives. When discussing how collusion between manufacturers is sustained or deterred, it is noteworthy that four classes of equilibrium outcomes might occur:

1. Platforms set s_{AC}^{**} , and manufacturers sustain collusion.
2. Platforms set a pair of revenue-sharing rules such that (i) collusion is sustainable, (ii) the pair of rules satisfies the mutual best response property in the set of pairs of revenue-sharing rules that makes collusion sustainable, and (iii) deviation in a way that makes collusion unsustainable is not profitable.
3. Platforms set a pair of revenue-sharing rules such that (i) collusion is unsustainable, (ii) the pair of rules satisfies the mutual best response property in the set of a pair of revenue-sharing rules that makes collusion unsustainable, and (iii) deviation in a way that makes collusion sustainable is not profitable.
4. Platforms set s_{AN}^{**} . Then manufacturers play the stage-game equilibrium strategies.

In the monopoly platform case, we show that the agency contract neither facilitates nor obstructs upstream collusion. In other words, the critical discount factor under the agency contract is the same as that under the wholesale contract. This result stems from $\Pi_{AC}^* < \Pi_{AN}^*$ of Lemma 2. For a case with competing platforms, however, we obtained $\Pi_{AC}^{**} > \Pi_{AN}^{**}$ in Lemma 6 with the static analysis of the stage-game. Additionally in the repeated interactions, we derive the following contrasting result.

Lemma 9. *The following statements hold.*

- (i) $\Pi_{dA}^C(s, s) > \Pi_{dA}^N(s, s)$ for any $s \in (0, 1)$.
- (ii) $\Pi_{dA}^C(s_{AC}^{**}, s_{AC}^{**}) > \Pi_{dA}^N(s_{AN}^{**}, s_{AN}^{**})$.

Lemma 9 (i) states that, for any s symmetrically set by the platforms, the profit of the platforms is higher when the manufacturers collude. Lemma 9 (ii) shows that the platforms benefit from the manufacturers' collusion even if they endogenize their revenue-sharing rules. In total, the platforms are better off with upstream collusion, as compared to when they play the stage-game Nash equilibrium, which is a sharp contrast with the monopoly platform case.

Some intuition can be provided for why the presence of platform competition drastically alters the attitude of platforms towards upstream collusion. In the monopoly platform case, when manufacturers play the stage-game Nash equilibrium, the platform can charge a higher revenue-sharing rule and then receive greater profit. However, the presence of platform competition would change the favorable situation as follows. Fixing s_1 and s_2 , then compared to the collusive equilibrium, gross revenues generated in the channel are lower under the stage-game Nash equilibrium, which induces both the platforms to lower their revenue-sharing rules. Consequently, because of strategic complementarity, the platforms become involved in fierce competition. By contrast, when the manufacturers collude, the reverse happens. That is, upstream collusion mitigates the platform competing.¹⁷

We obtain the following result for the critical discount factors.

Proposition 5. *In the model with two competing platforms, $\delta_A^{**} < \delta_W^{**}$ is satisfied.*

Proof. See the Appendix. □

Proposition 5 shows that the critical discount factor is lower under the agency contract than under the wholesale contract when two platforms mutually compete. Consequently, the presence of platform competition markedly alters our result derived for the monopoly platform case. That is, with platform competition, the agency contract facilitates upstream collusion.

The intuition underlying this important result is the following. First, in the presence of a competitor, each platform alone cannot perfectly control the manufacturers' actions by manipulating its revenue-sharing rule. Second, as stated in Lemma 9, for a fixed revenue-sharing rule, collusion among manufacturers generates greater channel revenue. The first effect renders platforms as *unable* to induce the manufacturers to play the stage-game Nash equilibrium. The second effect leads platforms to *favor* upstream collusion. Together, these effects make the agency contract facilitate upstream collusion.

Furthermore, this result might be related to the high commission rate of 30% imposed by Apple and Google in the mobile application market. In fact, this attention-getting antitrust case reached the U.S. Supreme Court.

¹⁷It is worth noting that this coordination effect between platforms is special to the ad-valorem characteristics of the revenue-sharing rule. If the platform charges a per-unit commission fee, manufacturers' collusion decreases the amount of transactions, resulting in a decrease in the equilibrium commission fee.

4 Discussion

In this section, we discuss several issues missing from the main analysis.

4.1 Fixed revenue-sharing rule

The main analysis shows that the agency contract facilitates upstream collusion in the market with two competing platforms, but it does not affect the sustainability of upstream collusion in the monopoly platform market. In this subsection, we discuss the robustness of the result derived in the monopoly platform case. We have assumed that the monopoly platform chooses a revenue-sharing rule at the beginning of every stage-game. Consequently, in the punishment phase, after one manufacturer deviates from the collusion strategy, not only the other manufacturers but also the platforms change their strategies. However, in reality, platforms tend to commit to a fixed revenue-sharing rule. Therefore, this subsection presents examination of the case in which the monopoly platform sets a revenue-sharing rule first; then manufacturers play an infinitely repeated game.¹⁸

First, inspecting Proposition 1 and inequality (17), under the Nash-reversion grim-trigger strategies, it would follow that the critical discount factor is the same for the two contracts, even if the revenue-sharing rule is fixed.

In this respect, here we try the other strategy, which is the optimal punishment strategy in the sense of Abreu (1986). In the following discussion, we show that the critical discount factor is smaller under the agency contract than under the wholesale contract if manufacturers follow the symmetric optimal punishment strategy. This result implies that, by tailoring a sophisticated punishment scheme, manufacturers can sustain price-fixing cartels easier under the agency contract.

Let $\hat{\delta}_W$ be the critical discount factor above which the joint profit maximization is sustainable by optimal punishment under the wholesale contract. Similarly, let $\hat{\delta}_A(s)$ be the critical discount factor above which the joint profit maximization is sustainable by optimal punishment under the agency contract with a fixed revenue-sharing rule s . Consequently, we obtain the following result.

Proposition 6. *For any fixed $s > 0$, it holds that $\hat{\delta}_A(s) < \hat{\delta}_W$.*

Proof. See the Appendix.

Let us briefly provide the related intuition. Under the wholesale contract, for any wholesale price, the platform makes a decision about how many goods to purchase from manufacturers in an effort to maximize its own profit. As a result, the platform derives a demand function that is similar to that of the representative consumer. In this situation, the sustainability of upstream collusion coincides with that under the agency contract without revenue sharing (i.e., $s = 0$).

Next, given the argument above, we consider the agency contract with a positive revenue-sharing rule $s > 0$. There are two effects of revenue-sharing rule on the critical discount factor under the

¹⁸Huang (2017) studies the model with the similar timeline to investigate downstream collusion. She assumes that, before starting an infinitely repeated game, a monopoly upstream supplier offers a two-part tariff contract to retailers. Subsequently, downstream retailers engage repeatedly and infinitely in collusive actions.

optimal punishment. First, the existence of a positive revenue-sharing rule makes deviation from the collusion unattractive because the revenue-sharing rule scales down the manufacturers' profits from gross sales revenues, which is maximized by the deviation. Second, in the punishment phase, the effect of revenue-sharing on the profit of manufacturers is small relative to that in the collusive phase because sales revenues obtained in the punishment phase are already small. This effect makes the collusion difficult to sustain. Overall, the first effect dominates the second one; the agency contract makes the collusion more sustainable.

4.2 Observability of actions

For the main analysis, we have assumed perfect monitoring, i.e., manufacturers can observe all actions of the others. In reality, however, there might be situations in which wholesale prices set by other manufacturers are not publicly observable, although the retail prices are easier to observe. It is well-known in the literature of collusion under imperfect monitoring that the more difficult it is to observe each player's action, the more difficult it is to sustain the collusion.¹⁹ Consequently, under the wholesale contract, one would have the higher critical discount factor if the wholesale prices were not publicly observable. That fact enhances the robustness of our results such that the agency contract might be more likely to facilitate collusion among the manufacturers.

5 Conclusion

This paper addresses the important issue of the sustainability of a digital cartel. To address the relation between cartel sustainability and the contract form, we develop a stylized model of the infinitely repeated game. We obtain and compare the critical discount factors, above which the price cartel among upstream manufacturers can be sustainable, under a wholesale contract and an agency contract. The central message from our study is that the agency contract neither facilitates nor obstructs upstream collusion in the case of monopoly platform, although it facilitates upstream collusion with the platform competition.

Our results are expected to contribute to the literature by providing important policy implications. The agency contract is not necessarily *per se illegal*, but competition authorities must be more concerned about it when several platforms compete in the market. As compared to the wholesale contract, under the agency contract, the monopoly platform is apparently able to affect upstream collusion by manipulating the revenue-sharing rule. However, our analysis demonstrates that the platform has no incentive to manipulate the formation of price cartel among upstream manufacturers. By contrast, in the market with competing platforms, accepting upstream collusion can ease the platform competition on their revenue-sharing rules, which makes the platforms facilitate the formation of upstream collusion. Consequently, with the platform competition, the agency contract facilitates upstream collusion.

¹⁹See Jullien and Rey (2007) for this argument applied to retail-price maintenance.

We conclude by describing the limitations of our model and by discussing potential avenues for future research. First, we do not examine the effect of Most-Favored-Nation (MFN) clauses. In the real case of the e-book cartel, however, the clause played a central role in the proceedings along with the agency agreements.²⁰ Secondly, we assume that, in the stage-game of the wholesale contract, manufacturers move before the platform(s). In the e-book market before Apple's entry, Amazon had set retail prices of most e-books at \$9.99 and had committed to such a pricing policy. It might be valuable to analyze the wholesale contract with realistic timing of moves, where platforms move before manufacturers.²¹ Lastly, in the model of platform competition, we compare the sustainability of collusion between two symmetric situations where both platforms impose the same contract (wholesale or agency) on their manufacturers. As typically observed in Amazon's behavior in the e-book market, however, platforms can choose their contract forms strategically. In this regard, it would be interesting to consider the platforms' endogenous choices about which contract to select, and to confirm whether and when the agency contract is adopted in equilibrium. These extensions require more complex analysis, which is beyond the scope of this paper.

Appendix A: Proofs for monopoly platform case

Proof of Lemma 2.

First, we show $s_{AN}^* > s_{AC}^*$. We focus on the first-order conditions with respect to s , i.e., equations (8) and (12). Consider that the revenue-sharing rule is set at the one in the collusive case (i.e., $s = s_{AC}^*$). The derivative of platform's profit under the punishment phase is computed by

$$\frac{\partial \Pi_{AN}(s_{AC}^*)}{\partial s} = \frac{n\beta}{(\beta + \sigma)^2} \cdot \left[\begin{aligned} & \left(\alpha + \beta \frac{c}{1-s_{AC}^*} \right) \left(\alpha - \sigma \frac{c}{1-s_{AC}^*} \right) \\ & + \frac{s_{AC}^* c}{(1-s_{AC}^*)^2} \left\{ \alpha(n-1)\gamma - 2\beta\sigma \frac{c}{1-s_{AC}^*} \right\} \end{aligned} \right] \quad (\text{A.1})$$

$$= \frac{n\beta}{(\beta + \sigma)^2} \cdot \left[\alpha^2 + \frac{\alpha(\beta - \sigma)c}{(1-s_{AC}^*)^2} - \frac{(1+s_{AC}^*)\beta\sigma c^2}{(1-s_{AC}^*)^3} \right] \quad (\text{A.2})$$

$$= \frac{n\beta}{(\beta + \sigma)^2} \cdot \left[\frac{(1+s_{AC}^*)\sigma^2 c^2}{(1-s_{AC}^*)^3} + \frac{\alpha(\beta - \sigma)c}{(1-s_{AC}^*)^2} - \frac{(1+s_{AC}^*)\beta\sigma c^2}{(1-s_{AC}^*)^3} \right] \quad (\text{A.3})$$

$$= \frac{n\beta}{(\beta + \sigma)^2} \cdot \frac{(\beta - \sigma)c}{(1-s_{AC}^*)^3} \cdot \{(1-s_{AC}^*)\alpha - (1+s_{AC}^*)\sigma c\}, \quad (\text{A.4})$$

where the third equality follows from equation (12).

If $\partial \Pi_{AN}(s_{AC}^*)/\partial s > 0$, then $s_{AN}^* > s_{AC}^*$. Therefore, we suffice to show $\partial \Pi_{AN}(s_{AC}^*)/\partial s > 0$, which is equivalent to $s_{AC}^* < (\alpha - \sigma c)/(\alpha + \sigma c)$. By looking at the derivative of platform's profit

²⁰For example, Boik and Corts (2016), Johansen and Vergé (2017), Johnson (2017), and Maruyama and Zennyo (2018) are noteworthy for providing analyses of so-called a Most-Favored-Customer clause or a price parity clause.

²¹Timing of moves of this type has been analyzed in Marketing Science and Operations Research, and so-called *retailer-stackelberg* game (e.g., Choi, 1991).

under collusion with respect to s at $s = (\alpha - \sigma c)/(\alpha + \sigma c)$, we have

$$\frac{\partial \Pi_{AC}}{\partial s} \left(s = \frac{\alpha - \sigma c}{\alpha + \sigma c} \right) = -\frac{n}{4\sigma} \cdot \frac{\alpha(\alpha - \sigma c)^2}{4\sigma c} < 0 \iff s_{AC}^* < \frac{\alpha - \sigma c}{\alpha + \sigma c}, \quad (\text{A.5})$$

which was what we wanted.

Second, we show $p_{AN}^* < p_{AC}^*$. It holds that $p_{AN}(s) < p_{AC}(s)$ for all $s \in (0, 1 - \sigma c/\alpha)$. Since both $p_{AN}(s)$ and $p_{AC}(s)$ are increasing function in s , there exists $s' > s_{AC}^*$ such that $p_{AN}(s') = p_{AC}(s_{AC}^*)$ holds. Substituting $s = s'$ into $\partial \Pi_{AN}(s)/\partial s$ and then simplifying it with equation (12), we have

$$\frac{\partial \Pi_{AN}(s')}{\partial s} = -\frac{\beta^2 - \sigma^2}{4(1 - s_{AC}^*)^3 \beta^2 \sigma} \left[\begin{aligned} &(1 - s_{AC}^*)^2 \alpha^2 (\beta - \sigma) - (\beta + \sigma) \sigma^2 c^2 \\ &+ 2(1 - s_{AC}^*) \alpha (s_{AC}^* \beta + \sigma) \sigma c \end{aligned} \right] < 0, \quad (\text{A.6})$$

which implies that $s_{AN}^* < s'$. Because $p_{AN}(s)$ is increasing function, it follows that $p_{AC}(s_{AC}^*) = p_{AN}(s') > p_{AN}(s_{AN}^*)$. The higher retail price under the collusive equilibrium directly leads the lower demand, consumer surplus, and social welfare (i.e., $q_{AN}^* > q_{AC}^*$, $CS_{AN}^* > CS_{AC}^*$, and $SW_{AN}^* > SW_{AC}^*$).

Third, we show $\pi_{AN}^* < \pi_{AC}^*$. It holds that $\pi_{AN}(s) < \pi_{AC}(s)$ for all $s \in (0, 1 - \sigma c/\alpha)$. The manufacturers' profit is decreasing in s . Because $s_{AN}^* > s_{AC}^*$, it follows that $\pi_{AN}(s_{AN}^*) < \pi_{AC}(s_{AC}^*)$.

Lastly, we show $\Pi_{AC}^* \leq \Pi_{AN}^*$. As above, we consider $s = s' > s_{AC}^*$ such that $p_{AN}(s') = p_{AC}(s_{AC}^*)$. The same retail price leads the same total output, which also implies the same gross revenue in the channel. Because s' is larger than s_{AC}^* , the platform gains the larger share of the gross revenue, that is, $\Pi_{AC}(s_{AC}^*) < \Pi_{AN}(s')$. Finally, from the definition of s_{AN}^* , the platform can obtain the greater profit by charging s_{AN}^* than s' , that is, $\Pi_{AN}(s') \leq \Pi_{AN}(s_{AN}^*)$. Therefore, it holds that $\Pi_{AN}^* > \Pi_{AC}^*$. \square

Proof of Lemma 3.

First, we can show that $p_{WC}^* > p_{AC}^*$ and $q_{WC}^* < q_{AC}^*$ by simple calculations, which in turn imply that $CS_{WC}^* < CS_{AC}^*$ and $SW_{WC}^* < SW_{AC}^*$.

Next, we show that $\pi_{WC}^* > \pi_{AC}^*$ holds. There exists a unique $s'' \in (0, 1 - \sigma c/\alpha)$ such that $\pi_{WC}^* = \pi_{AC}(s'')$. Substituting $s = s''$ into $\partial \Pi_{AC}(s)/\partial s$, we have $\partial \Pi_{AC}(s'')/\partial s > 0$. Since $\Pi_{AC}(s)$ is concave, it holds that $s'' < s_{AC}^*$. Moreover, because $\pi_{AC}(s)$ is decreasing in s , it holds that $\pi_{AC}(s'') > \pi_{AC}(s_{AC}^*)$. In sum, we can derive that $\pi_{WC}^* = \pi_{AC}(s'') > \pi_{AC}(s_{AC}^*) = \pi_{AC}^*$ holds.

Lastly, we show that $\Pi_{WC}^* < \Pi_{AC}^*$ holds. Set s' such that $s' = (p_{WC}^* - w_{WC}^*)/p_{WC}^*$. If $p_{AC}(s') < p_{WC}^*$ holds, then we have

$$p_{WC}^* q(p_{WC}^*) < p_{AC}(s') q(p_{AC}(s')) \quad (\text{A.7})$$

by the concavity of $pq(p)$ in p and the inequality $\arg \max_p pq(p) < p_{AC}(s') < p_{WC}^*$, where $q(p) \equiv q_i(p, \dots, p)$ for $i = 1, \dots, n$. This in turn implies that

$$\Pi_{AC}^* = \max_s \Pi_{AC}(s) \geq \Pi_{AC}(s) = \frac{p_{WC}^* - w_{WC}^*}{p_{WC}^*} p_{AC}(s') q(p_{AC}(s')) > \Pi_{WC}^*. \quad (\text{A.8})$$

What remains to be shown is that $p_{AC}(s') < p_{WC}^*$. By the fact that $p_{WC}^* = (3\alpha + \sigma c)/4\sigma$, $w_{WC}^* = \frac{\alpha}{2\sigma} + \frac{c}{2}$, and $p_{AC}(s) = \{\alpha + \sigma c/(1-s)\}/2\sigma$, we have

$$p_{WC}^* - p_{AC}(s') = \frac{\alpha(\alpha - \sigma c)}{4\sigma(\alpha + \sigma c)} > 0. \quad (\text{A.9})$$

Thus, we have $p_{AC}(s') < p_{WC}^*$, which completes the proof. \square

Proof of Lemma 4.

(i) First, $\delta_A(s, s^*)$ is increasing in s^* simply because $\pi_{AC}(s^*)$ is decreasing. Second, $\delta_A(s, s^*)$ is decreasing in s because

$$\frac{\partial \pi_{AD}(s)}{\partial s} - \frac{\partial \pi_{AC}(s)}{\partial s} = \frac{\partial}{\partial s} \left\{ (1-s) \left(\alpha - \sigma \frac{c}{1-s} \right)^2 \right\} \left\{ \frac{(\beta + \sigma)^2}{16\beta\sigma^2} - \frac{1}{4\sigma} \right\} < 0. \quad (\text{A.10})$$

(ii) Formally, $\tilde{\delta}_A(s)$ is computed as follows.

$$\tilde{\delta}_A(s) = \delta_A(s, s) = \frac{\pi_{AD}(s) - \pi_{AC}(s)}{\pi_{AD}(s) - \pi_{AN}^*} \quad (\text{A.11})$$

The derivative of $\tilde{\delta}_A(s)$ with respect to s is given by

$$\frac{\partial \tilde{\delta}_A(s)}{\partial s} = \frac{\left(\frac{\partial \pi_{AD}(s)}{\partial s} - \frac{\partial \pi_{AC}(s)}{\partial s} \right) \{ \pi_{AD}(s) - \pi_{AN}^* \} - \{ \pi_{AD}(s) - \pi_{AC}(s) \} \frac{\partial \pi_{AD}(s)}{\partial s}}{\{ \pi_{AD}(s) - \pi_{AN}^* \}^2} \quad (\text{A.12})$$

$$= \frac{\frac{1}{4\sigma} \left(\alpha + \sigma \frac{c}{1-s} \right) \left(\alpha - \sigma \frac{c}{1-s} \right) \frac{(\beta - \sigma)^2}{4\beta\sigma} \pi_{AN}^*}{\{ \pi_{AD}(s) - \pi_{AN}^* \}^2}, \quad (\text{A.13})$$

which is greater than 0. Therefore, $\tilde{\delta}_A(s)$ is an increasing function.

(iii) Substituting $s = s_{AN}^*$ into $\tilde{\delta}_A(s)$ directly yields $\tilde{\delta}_A(s_{AN}^*) = \delta_W^*$. \square

Proof of Proposition 2.

First, we prove that choosing $s^* = s_{AN}^*$ at every period is the platform-preferred equilibrium if and only if $\delta < \tilde{\delta}_A(s_{AN}^*)$.

- *Proof of if*

Suppose that $\delta < \tilde{\delta}_A(s_{AN}^*)$. From the definition of $\tilde{\delta}_A(\cdot)$, if the revenue-sharing rule is set at $s = s_{AN}^*$, the collusion among manufacturers cannot be sustained for $\delta \in [0, \tilde{\delta}_A(s_{AN}^*)]$. In addition, from Lemma 2, the platform has no incentive to choose s other than s_{AN}^* . Thus, choosing $s^* = s_{AN}^*$ at every period is the platform-preferred equilibrium if $\delta < \tilde{\delta}_A(s_{AN}^*)$. Moreover, this is a unique equilibrium.

- Proof of *only if*

We show a contraposition, that is, if $\delta \geq \tilde{\delta}_A(s_{AN}^*)$, then setting $s = s_{AN}^*$ every period is never equilibrium. Suppose that, to derive a contradiction, the platform sets s_{AN}^* at every period. Then, the manufacturers can sustain their joint profit maximizing collusion as long as $s \geq \tilde{s}(\delta, s_{AN}^*)$. Now, it holds that $s_{AN}^* \geq \tilde{s}(\delta, s_{AN}^*)$ because $\delta \geq \tilde{\delta}_A(s_{AN}^*)$. From $s_{AC}^* < s_{AN}^*$, the platform can increase its profit by reducing s below s_{AN}^* , a contradiction. Therefore, choosing $s^* = s_{AN}^*$ at every period is the platform-preferred equilibrium *only if* $\delta < \tilde{\delta}_A(s_{AN}^*)$.

Second, we prove that choosing $s^* = s_{AC}^*$ at every period is the platform-preferred equilibrium if and only if $\delta \geq \tilde{\delta}_A(s_{AN}^*)$.

- Proof of *if*

Suppose that $\delta \geq \tilde{\delta}_A(s_{AN}^*)$. Since $\tilde{\delta}_A(s)$ is increasing in s (from Lemma 4 (ii)) and $s_{AC}^* < s_{AN}^*$ (from Lemma 2), it holds that $\tilde{\delta}_A(s_{AC}^*) < \tilde{\delta}_A(s_{AN}^*)$. Thus, for $\delta \in [\tilde{\delta}_A(s_{AN}^*), 1)$, the manufacturers can sustain their joint profit maximizing collusion. We check that the platform has no incentive to set s other than s_C in each period. To do so, it suffices to verify that the platform cannot obtain the greater profit by deterring collusion by setting $s = \tilde{s}(\delta, s_{AC}^*) < s_C$, i.e.,

$$\Pi_{AN}(\tilde{s}(\delta, s_{AC}^*)) < \Pi_{AC}(s_{AC}^*), \quad (\text{A.14})$$

which holds. In addition, since there is no pure strategy equilibrium in the class of strategies considered in our setting. Therefore, choosing $s^* = s_{AC}^*$ at every period is the platform-preferred equilibrium *if* $\delta \geq \tilde{\delta}_A(s_{AN}^*)$.

- Proof of *only if*

We show a contraposition, that is, if $\delta < \tilde{\delta}_A(s_{AN}^*)$, then choosing $s = s_{AC}^*$ at every period is never equilibrium. As shown above, if $\delta < \tilde{\delta}_A(s_{AN}^*)$, choosing $s^* = s_{AN}^*$ at every period is the *unique* platform-preferred equilibrium. That is, the contraposition holds. Thus, choosing $s^* = s_{AC}^*$ at every period is the platform-preferred equilibrium *only if* $\delta \geq \tilde{\delta}_A(s_{AN}^*)$.

In sum, if $\delta < \tilde{\delta}_A(s_{AN}^*)$, then the platform chooses $s^* = s_{AN}^*$, which deter the collusion among manufacturers. In contrast, if $\delta \geq \tilde{\delta}_A(s_{AN}^*)$, the platform chooses $s^* = s_{AC}^*$, where the manufacturers can sustain their joint profit maximizing collusion. Therefore, $\tilde{\delta}_A(s_{AN}^*)$ is the critical discount factor under the agency contract, that is, $\delta_A^* = \tilde{\delta}_A(s_{AN}^*)$. \square

Appendix B: Proofs for competing platforms case

Before giving proofs for Lemma 8, Lemma 9, and Proposition 5, let us take a closer look at the collusion phase under the agency contract. The first-order condition for joint profit maximization given s_1 and s_2 is given by

$$\frac{\partial(\pi^1 + \pi^2)}{\partial p_{dA}^u} = (1-s_d) \left(q_d^u + p_{dA}^u \frac{\partial q_d^u}{\partial p_d^u} + p_d^{-u} \frac{\partial q_d^{-u}}{\partial p_d^u} \right) + (1-s_{-d}) \left(p_{-dA}^u \frac{\partial q_{-d}^u}{\partial p_d^u} + p_d^{-u} \frac{\partial q_{-d}^{-u}}{\partial p_d^u} \right) = 0. \quad (\text{B.1})$$

Invoking symmetry, $p_d^u = p_d^{-u}$, we obtain

$$p_d(s_d, s_{-d}) = \frac{(1-\lambda) \left[2 + \lambda \left(1 + \frac{1-s_{-d}}{1-s_d} \right) \right]}{4 - \lambda^2 \left(1 + \frac{1-s_{-d}}{1-s_d} \right) \left(1 + \frac{1-s_d}{1-s_{-d}} \right)} \quad \text{and} \quad q_d(s_d, s_{-d}) = \frac{(1-\lambda) - p_d + \lambda p_{-d}}{(1-\lambda^2)(1+\mu)}. \quad (\text{B.2})$$

Assuming $s_1 = s_2 = s$, we obtain the resulting price, quantities, profit of manufacturers, and profit of platforms as follows.

$$p_{AC}(s, s) = \frac{1}{2} \quad (\text{B.3})$$

$$q_{AC}(s, s) = \frac{1}{2(1+\lambda)(1+\mu)} \quad (\text{B.4})$$

$$\pi_{AC}(s, s) = \frac{1-s}{2(1+\lambda)(1+\mu)} \quad (\text{B.5})$$

$$\Pi_{AC}(s, s) = \frac{s}{2(1+\lambda)(1+\mu)} \quad (\text{B.6})$$

Finally, in an analogous way to the case of wholesale contract, the deviation from the joint profit maximization yields $p_{AD}(s, s) = (2-\mu)/4$ and $\pi_{AD}(s, s) = (1-s)(2-\mu)^2/\{8(1+\lambda)(1-\mu^2)\}$. Using these values, the critical discount factor given $s_1 = s_2 = s^{**} = s$ can be obtained as follows.

$$\delta_A(s, s, s) = \frac{\pi_{AD}(s, s) - \pi_{AC}(s, s)}{\pi_{AD}(s, s) - \pi_{AN}^{**}} = \frac{(1-s) \frac{(2-\mu)^2}{8(1+\lambda)(1-\mu^2)} - (1-s) \frac{1}{2(1+\lambda)(1+\mu)}}{(1-s) \frac{(2-\mu)^2}{8(1+\lambda)(1-\mu^2)} - (1-s_{AN}^{**}) \frac{2(1-\mu)}{(1+\lambda)(1+\mu)(2-\mu)^2}} \quad (\text{B.7})$$

Thus, $\delta_A(s, s, s) < \delta_W^{**}$ if and only if $s < s_{AN}^{**}$.

Using above preliminary results, we show Lemma 8, Lemma 9, and Proposition 5.

Proof of Lemma 8

(i) $\delta_A(s, s, s)$ can be rewritten by

$$\delta_A(s, s, s) = \frac{\frac{(2-\mu)^2}{8(1+\lambda)(1-\mu^2)} - \frac{1}{2[(1+\lambda)(1+\mu)]}}{\frac{(2-\mu)^2}{8(1+\lambda)(1-\mu^2)} - \frac{(1-s_{AN}^{**})}{(1-s)} \frac{2(1-\mu)}{(1+\lambda)(1+\mu)(2-\mu)^2}}, \quad (\text{B.8})$$

which is increasing in s .

(ii) Using envelope theorem, we obtain

$$\frac{\partial \pi_{AC}(s, s)}{\partial s_d} = -p_d^C q_d^C = -\frac{1}{4(1+\lambda)(1+\mu)}, \quad (\text{B.9})$$

$$\frac{\partial \pi_{AD}(s, s)}{\partial s_d} = -p_d^D q_d^D + \left(\frac{\partial p_d^C}{\partial s_d} + \frac{\partial p_{-d}^C}{\partial s_d} \right) (1-s) p_d^D \left(\frac{\partial (q_d^u + q_{-d}^u)}{\partial p_d^{-u}} \right) \quad (\text{B.10})$$

$$= \frac{(2-\mu)[8\mu\lambda - (2-\mu)(1+\lambda)]}{16(1+\lambda)^2(1-\mu^2)}. \quad (\text{B.11})$$

Thus, applying the implicit function theorem, we have

$$\text{sign} \left(\frac{\partial \delta_A(s, s, s^{**})}{\partial s_d} \right) = \text{sign} \left(\frac{\partial \pi_{AD}(s, s)}{\partial s_d} - \frac{\partial \pi_{AC}(s, s)}{\partial s_d} \right) = \mu \{16\lambda - \mu(1 + 9\lambda)\} \quad (\text{B.12})$$

Under the assumption that $\lambda \geq \mu/(16 - 9\mu)$, we have that $\partial(\pi_{AD}(s, s) - \pi_{AC}(s, s))/\partial s_d > 0$, and thus, $\partial \delta_A(s, s, s^{**})/\partial s_d > 0$.

In addition, $\delta_A(s, s, s^*) \in (0, 1)$ implies that $\pi_{AD}(s, s) - \pi_{AN}^{**} > 0$ and $\pi_{AD}(s, s) - \pi_{AN}^{**} > \pi_{AD}(s, s) - \pi_{AC}(s, s) \geq 0$. When $\partial \pi_{AD}(s, s)/\partial s_d < 0$, all the terms in the numerator of $\partial \delta_A(s, s, s^*)/\partial s_d$ are nonnegative and one of them is strictly positive. Otherwise, when $\partial \pi_{AD}(s, s)/\partial s_d > 0$, the following string of inequalities hold:

$$\begin{aligned} & \left(\frac{\partial \pi_{AD}(s, s)}{\partial s_d} - \frac{\partial \pi_{AC}(s, s)}{\partial s_d} \right) \{ \pi_{AD}(s, s) - \pi_{AN}^{**} \} - \{ \pi_{AD}(s, s) - \pi_{AC}(s, s) \} \frac{\partial \pi_{AD}(s, s)}{\partial s_d} \\ & > - \frac{\partial \pi_{AC}(s, s)}{\partial s_d} (\pi_{AD}(s, s) - \pi_{AN}^{**}) > 0. \end{aligned} \quad (\text{B.13})$$

Thus, in both cases, $\partial \delta_A(s, s, s^{**})/\partial s_d > 0$ holds. \square

Proof of Lemma 9

(i) When $s_1 = s_2 = s$, manufacturers set their retail prices at $p_{AC}(s, s) = 1/2$, which is independent of s . Thus, we suffice to check $p_{AC}^{**} q_{AC}^{**} > p_{AN}^{**} q_{AN}^{**}$, which can easily verified from the stage-game Nash equilibrium and the collusive equilibrium under the agency contract that are summarized at Table 4.

(ii) It holds that $s_{AC}^{**} > s_{AN}^{**}$. Thus, the following string of inequalities shows what we wanted.

$$\Pi_{dA}^C(s_{AC}^{**}, s_{AC}^{**}) = 2s_{AC}^{**} p_{AC}^{**} q_{AC}^{**} > 2s_{AN}^{**} p_{AC}^{**} q_{AC}^{**} = \Pi_{dA}^C(s_{AN}^{**}, s_{AN}^{**}) > \Pi_{dA}^N(s_{AN}^{**}, s_{AN}^{**}) \quad (\text{B.14})$$

\square

Proof of Proposition 5

Instead of showing this proposition, we first show the following proposition. Then, we show that the same result as the main text holds:

Proposition 7. *For all $\delta \geq \delta_W^{**} - \varepsilon$ with small $\varepsilon > 0$, joint profit maximization among manufacturers is sustainable in the platform-preferred equilibrium under the agency contract.*

Proof. We consider the condition for s to be a symmetric equilibrium revenue-sharing rule that sustains the collusion. There are two conditions; (1) at $(s_1, s_2, s^{**}) = (s, s, s)$, joint profit maximization is supported by the manufacturers' Nash-reversion trigger strategies, and for each (s_1, s_2) , the manufacturers maximize their joint profit if possible and follows stage-game Nash equilibrium otherwise, and (2) the platforms have incentives to choose s in each period.

(i) For joint profit maximization to be sustainable, we must have $\delta > \delta_A(s, s, s)$.

(ii) To see whether the platforms have incentives to choose s in each period, we must check one-shot deviation incentives. Suppose that the platform 1 chooses $s_1 \neq s$. Then, the joint profit maximization among manufacturers is sustainable if and only if

$$\pi_{AD}(s_1, s) - \pi_{AC}(s_1, s) \leq \frac{\delta}{1-\delta} (\pi_{AC}(s, s) - \pi_{AN}^{**}). \quad (\text{B.15})$$

Thus, if s satisfies $\delta_A(s, s, s) = \delta$, then the joint profit maximization is sustainable if and only if $s_1 \leq s$ by Assumption 1. Therefore, if the platform 1 deviates so that $s_1 > s$, the manufacturers follow the stage-game Nash equilibrium strategies, and if the platform 1 deviates so that $s_1 < s$, the manufacturers maximize their joint profit.

First, if $\delta > \delta_A(s_{AC}^{**}, s_{AC}^{**}, s_{AC}^{**})$, a pair of $(s_{AC}^{**}, s_{AC}^{**})$ is supported as the equilibrium, and thus the joint profit maximization by manufacturers is sustained.

Next, when $\delta_A(s_{AN}^{**}, s_{AN}^{**}, s_{AN}^{**}) < \delta \leq \delta_A(s_{AC}^{**}, s_{AC}^{**}, s_{AC}^{**})$, there is some $s \in (s_{AN}^{**}, s_{AC}^{**}]$ such that $\delta = \delta_A(s, s, s)$. At such s , the joint profit maximization is sustainable. Since $s < s_{AC}^{**}$, each platform has no incentive to decrease s . By Assumption 1, a further increase in s_d makes joint profit maximization unsustainable, but this is not optimal for each platform because it leads to a discontinuous drop in the profit (i.e., $\Pi_{AC}(s, s) > \Pi_{AN}(s, s)$ for any s).

Finally, we show that $\delta_A^{**} < \delta_W^{**}$. Take a small $\varepsilon > 0$ and let $\delta = \delta_W^{**} - \varepsilon$. We show that $s < s_{AN}^{**}$ such that $\delta_A(s, s, s) = \delta_W^{**} - \varepsilon$ can be supported as an equilibrium. On the one hand, following this strategy, platforms obtain $\Pi_{AC}(s, s)$. On the other hand, platform can obtain at most $\Pi_{AN}(BR^N(s), s)$ by making the joint profit maximization unsustainable, where $BR^N(s) := \arg \max_{s'} \Pi_{dA}^N(s', s)$. The difference between these values can be approximated by

$$\Pi_{AC}(s_{AN}^{**}, s_{AN}^{**}) - \Pi_{AN}(s_{AN}^{**}, s_{AN}^{**}) + o(\varepsilon), \quad (\text{B.16})$$

which is positive when ε is sufficiently small. Thus, the collusion is sustainable at the discount factor $\delta = \delta_W^{**} - \varepsilon$ for small $\varepsilon > 0$, which implies that $\delta_A^{**} < \delta_W^{**}$. \square

From this result, $\delta_A^{**} < \delta_W^{**}$ holds for $c = 0$. \square

Appendix C: Proofs for optimal punishment strategies

Proof of Proposition 6.

For each contract structure $K \in \{W, A\}$, let $\pi_{i,K}(a_i, a)$ be the profit of manufacturer i with action a_i , given the other manufacturers take the same action a . Define $\pi_K(a)$ and $\pi_K^D(a)$ by

$$\pi_K(a) = \pi_{i,K}(a, a), \quad \pi_K^D(a) = \arg \max_{a'} \pi_{i,K}(a', a). \quad (\text{C.1})$$

Let a_K^M be the action that maximizes joint profit for each contract $K \in \{W, A\}$.

Abreu (1986) shows that if there exist an action a^P and a discount factor $\hat{\delta}_K$ which satisfy the following system equations, then there exist no punishment rule which can sustain collusion and a^M for discount factor $\delta < \hat{\delta}_K$.²²

$$\pi_K^D(a^M) - \pi_K(a^M) = \delta_K(\pi_K(a^M) - \pi_K(a^P)), \quad (\text{C.2})$$

$$\pi_K^D(a^P) - \pi_K(a^P) = \delta_K(\pi_K(a^M) - \pi_K(a^P)). \quad (\text{C.3})$$

For $K = W$, we have

$$\pi_W(w) = \frac{1}{2}(w - c)(\alpha - \sigma w) \quad (\text{C.4})$$

$$\pi_W^D(w) = \frac{1}{8\beta}(\alpha + (1 - 2\beta)c + (n - 1)\gamma w)(a - c + (n - 1)\gamma w) \quad (\text{C.5})$$

$$\pi_W(w^M) = \frac{(\alpha - \sigma c)^2}{8\sigma} \quad (\text{C.6})$$

$$\pi_W^D(w^M) = \frac{(\beta + \sigma)^2(\alpha - \sigma c)^2}{32\beta\sigma^2} \quad (\text{C.7})$$

For $K = A$, we have

$$\pi_A(p) = (1 - s)(p - \kappa c)(\alpha - \sigma p) \quad (\text{C.8})$$

$$\pi_A^D(p) = (1 - s)\frac{1}{4\beta}(\alpha + (1 - 2\beta)\kappa c + (n - 1)\gamma p)(\alpha - \kappa c + (n - 1)\gamma p) \quad (\text{C.9})$$

$$\pi_A(p^M) = (1 - s)\frac{(\alpha - \sigma\kappa c)^2}{4\sigma} \quad (\text{C.10})$$

$$\pi_A^D(p^M) = (1 - s)\frac{(\beta + \sigma)^2(\alpha - \sigma\kappa c)^2}{16\beta\sigma^2}, \quad (\text{C.11})$$

where $\kappa := 1/(1 - s)$. If $s = 0$, then $\kappa = 1$, and the equations above becomes the same between wholesale contract and agency contract. Thus, we have the following lemma.

Lemma 10. *Let $\hat{\delta}_W$ and $\hat{\delta}_A(s)$ be the values of δ_K which satisfy the system of equations for each $K \in \{W, A\}$. Then, $\hat{\delta}_W = \hat{\delta}_A(0)$.*

Thus, it suffices to show that $\hat{\delta}_A(s)$ decreases with s . To this end, we show that δ_A decreases with κ .

Substituting the values of profits under agency contracts into the system of equation, we obtain

$$\hat{\delta}_A(s) = \frac{\frac{(\beta - \sigma)^2}{16\beta\sigma^2}}{\frac{1}{4\sigma} - \frac{(p(\kappa) - \kappa c)(\alpha - \sigma p(\kappa))}{(\alpha - \sigma\kappa c)^2}}, \quad (\text{C.12})$$

²²We do not consider the participation constraints of manufacturers. For the effects such constraints on the optimal punishment, see Lambertini and Sasaki (1999).

where

$$p(\kappa) = \frac{\alpha + \beta\kappa c - \sqrt{\frac{1}{4\sigma^2}(\beta - \sigma)^2(\alpha - \sigma\kappa c)^2 + (1 - \beta)^2\kappa^2 c^2}}{\beta + \sigma}. \quad (\text{C.13})$$

Then, an rudimental calculation yields the following result:

$$\begin{aligned} \text{sign} \left(\hat{\delta}'_A(s) \right) &= \text{sign} \left[\frac{d}{d\kappa} \left(\frac{(p(\kappa) - \kappa c)(\alpha - \sigma p(\kappa))}{(\alpha - \sigma\kappa c)^2} \right) \right] \\ &= \text{sign} \left[-(\alpha + \sigma\kappa c - 2\sigma p(\kappa)) \frac{(1 - \beta)^2\kappa c^2}{\sqrt{\frac{1}{4\sigma^2}(\beta - \sigma)^2(\alpha - \sigma\kappa c)^2 + (1 - \beta)^2\kappa^2 c^2}(\beta + \sigma)} \alpha \right] \\ &= - \end{aligned} \quad (\text{C.14})$$

since $p(\kappa) < p_A^M = (\alpha + \sigma\kappa c)/(2\sigma)$. □

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